Eleventh International Conference on Application of Mathematics in Technical and Natural Sciences
20–25 June 2019, Albena, Bulgaria

BOOK OF ABSTRACTS

Euro-American Consortium for Promoting the Application of Mathematics in Technical and Natural Sciences

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Direct Integration of Fourth Order Initial and Boundary Value Problems Using Nyström Type Methods

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Nyström type methods are widely used for the numerical integration of initial value problems (IVPs) in ordinary differential equations (ODEs). Specifically, they are extensively used for directly solving second order IVPs. Nevertheless, they are not normally used for the numerical integration of boundary value problems (BVPs). This paper focuses on the formulation of a family of block Nyström type methods (BNM $(A|A\ddagger, p)$) for the numerical solution of fourth order IVPs and BVPs, where $A|A\ddagger$ is the number of off-grid points and p is the order of the method. The family of BNM $(A|A\ddagger, p)$ is formulated from continuous schemes obtained via collocation and interpolation techniques and applied in a block-by-block manner as numerical integrators for fourth order ODEs. The convergence properties of the family of methods are discussed via zero-stability and consistency. Numerical examples are included and comparisons are made with existing methods in the literature.

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An Improved Barycentric Lagrange Double Interpolation for Solving Volterra Integral Equations of the Second Kind

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An improved and sophisticated version of Barycentric Lagrange interpolation with uniformly spaced interpolation nodes is established and applied to solve Volterra integral equations of the second kind. The given improved version of Barycentric Lagrange polynomial is obtained by redefining it in a matrix form in such a manner that the round-off errors of the calculations are remarkably minimized. Achieving this advantage, the presented method consists of three steps; the first is based on the interpolation of the given data function and the unknown function by using the improved Barycentric Lagrange polynomials of the same degree. In the second

step the kernel is interpolated twice with respect to both its variables by using the same interpolant polynomial of the same degree, so that it is transformed into a product of three matrices, where only one matrix is dependent on the given kernel. The importance of the third step may be summarized as follows: the interpolate unknown function is substituted twice into both sides of the considered integral equation. This enforcement provides the possibility to reduce the solution of Volterra equation into an equivalent algebraic linear system in matrix form without any need to apply collocation points. Moreover, seven illustrated numerical examples are solved. It turns out that, the obtained approximate solutions converge to the exact ones, which ensures the accuracy, efficiency, and authenticity of the presented method.

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Conjugate Gradient Method for Solving the Inverse Gravimetry Problem in Multilayered Medium: Parallel Implementation on GPU

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The paper is devoted to construction of the time efficient algorithm for solving the structural inverse gravimetry problem in the case of multilayered medium. The problem is in finding multiple interfaces between layers with different constant densities by known gravitational data. This problem is described by a nonlinear integral equation of the first kind. It is ill-posed problem. After discretization of the area and approximation of the integral operator, the problem is reduced to solving a system of nonlinear equations. An efficient method was constructed on the basis of the nonlinear conjugate gradient method. The algorithm uses the approximation of the Jacobian matrix of the integral operator based on dropping out the lesser elements and utilizing the Toeplitz-block-Toeplitz structure of the matrix. The parallel algorithm was implemented for the graphics processor (GPU) using CUDA technology. The structural gravimetry problem of reconstructing three surfaces using quasi-real data was solved.

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A Motion Correction of Stochastic Evolutionary Systems with Uncertainties

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We consider the evolutionary system in a Hilbert space H: $x(t) = S_t x_0 + \int_0^t S_{t-s}((Bu(s)+Cv(s))ds+dw(s))$, where $t \in [0,T]$, S_t is a semigroup with closed generator A with the dense domain $D(A)$ in H; B, C are linear operators from U, V, respectively; $u(t)$ is a control. Given a complete probability space $(\Omega, \mathcal{F}, P)$, an H-valued Wiener process $w(t)$ process with continuous paths is used in the system. The state $x(t)$ is unknown. There is a partition $0 = t_0 t_1 \dots t_N = T$ of $[0,T]$, $t_i - t_{i-1} = T/N = \delta$, for which vectors $y_i = Gx(t_{i-1}) + r_i + \eta_i$ at instants t_i are observed, where $G : H \to R^m$ is a linear operator, η_i are Gaussian mutually independent vectors which independent of $w(t)$. The state x_0 is independent of $w(t)$ and η_i. The uncertainties $\mathrm{E}x_0 = \bar{x}_0$, $v(\cdot)$, and r_i are restricted by constrains $\alpha^i = (\bar{x}_0, v^i(\cdot), r^i) \in \mathbf{V}^i$, where $\mathbf{V}^i$ is a weakly compact set in $H \times L_2([0,t_i],V) \times R^{mi}$ for every $i \in 1:N$. Similarly, the control $u(\cdot) \in \mathbf{U}$, where $\mathbf{U} \subset L_2([0,T],U)$. If $u(\cdot) \in \mathbf{U}$, then $\mathbf{U}^i$ is the set of all continuations of the section $u^i(\cdot)$ on $[0,t_i]$ to the segment $[t_i,T]$. If the signal till the t_i with known initial control $u_0^i(\cdot)$ is observed, then we minimize the terminal cost $\mathrm{E}\|x_N\|^2$ by controls $u(\cdot)$ depending of $y^i = \{y_1, \dots, y_i\}$ and $u(\cdot) \in \mathbf{U}^i$. The problem is solved by the minimax approach. First, the maximizer α_0^N in the problem $\mathrm{E}\|x_N\|^2 \to \max_{\alpha^N}$ is found under zero control. As $\mathrm{E}\|x_N\|^2 = \mathrm{trace}P_{N,i} + \mathrm{E}\|\hat{x}_{N,i}\|^2$, where $\hat{x}_{N,i} = \mathrm{E}(x_N|\sigma(y^i))$ is the conditional mean and $P_{N,i} = \mathrm{cov}(x_N - \hat{x}_{N,i}, x_N - \hat{x}_{N,i})$ is the covariance operator, we, secondly, solve the problem $\mathrm{E}\|\hat{x}_{N,i}\|^2 \to \min = J_i$ over $u(\cdot) \in \mathbf{U}^i$ for all $i \in 1:N$. Then we seek an optimal stopping time τ, depending on y^i, where $J_\tau \leq J_{\tau+1}$. The procedure may be repeated on the segment $[\tau, T]$. Some examples are considered, as well as some applications to physics and investments problem.

Acknowledgements. The work is supported by the Russian Foundation for Basic Research, Project No.18-01-00544-a.

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Some Qualitative Properties of the Solutions of Goodwin's Equation with Fixed Delay

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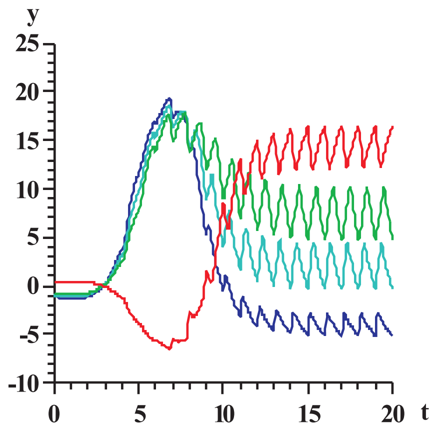
Investigations of the Goodwin equation [1] with a fixed delay

$$0.5\dot{y}(t) + 0.4y(t) = \begin{cases} -3, & \dot{y} < -1.5 \\ 2\dot{y}(t-\theta), & 4.5 > \dot{y} \geq -1.5 \\ 9, & \dot{y} \geq 4.5 \end{cases}$$

showed that the quality characteristics of the sawtooth oscillations (mean value of $y(t)$ and its amplitude) significantly depend on the shape of the initial function [2, 3], as shown in the figure for $\theta = 1$. Here is income in billions of dollars per year, t is time in years.



On the other hand, by using the method of equivalent linearization in [4] it was shown that the mean value and amplitude are constant. In this report, we show that the results of [4] are valid only for a certain shape of initial function and discuss the conditions of applicability of the method of equivalent linearization to the analysis of the properties of solutions of the Goodwin equation.

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Computer Simulation of Plasma Dynamics in Open Magnetic Systems

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The work is devoted to the numerical simulation of formation of diamagnetic "bubble" in an open magnetic trap [1]. This system makes possible to achieve the maximal plasma pressure for the effective plasma heating and confinement. We consider axially symmetrical magnetic trap with injection and subsequent ionization of neutral particle beams. The simulations of the ion beam - plasma interaction are based on a two-dimensional hybrid model for the quasi-neutral collisionless plasma, where the ions are described kinetically, and the electrons are considered as a fluid [2]. We use the particle-in-cells method (PIC) in the cylindrical coordinates and the finite-difference schemes for shifted grids. The high magnetic field values define the maximal time step due to the stability condition for the numerical scheme. Numerical experiments for plasma and ion beam parameters in accordance with the parameters of laboratory experiments required considerable computational resources and a parallel version of the developed algorithm. The parallelization we use is based on the domain decomposition, where each processor is responsible for its sub-domain and the particles in the domain. The problems of the algorithm accuracy, convergence and performance are considered. The results of numerical modeling of the plasma dynamics in the diamagnetic regime of the open plasma trap are presented. The computations were performed on the supercomputer Lomonosov (MSU, Moscow) and Siberian Supercomputer Center cluster (ICM&MG SB RAS, Novosibirsk).

Acknowledgements. The work was supported by the Russian Foundation for Basic Research, project No 18-29-21025.

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Equivalence Group for Generalized Kudryashov-Sinelshchikov Equations of Second Order

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The equivalence group for a class of nonlinear evolution partial differential equations, generalizing the second order Kudryashov-Sinelshchikov equation which describes pressure waves in liquid with bubbles, is calculated. Then a preliminary group classification of the equations in the considered class is obtained. After a simplification, based on the use of the found equivalence group, a complete group classification of these equations is carried out.

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Methodology for Optimizing the Control of Investment Projecting

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The tasks of improving investment projecting in the modern economy can be considered as necessary components of the control process of any enterprise and are precisely those tasks whose solution determines the financial policy of the economic structure. The development of technologies based on the optimization of investment projecting is a very relevant scientific problem. Investment projecting is designed to develop and implement not only the most profitable ways to generate income, but also to look for new profitable options for allocating free cash resources. In this paper, in order to develop an economic-mathematical model of the investment projecting control process, optimal control methods are used to solve applied problems in economics. The proposed methodology for optimizing the control of the investment projecting process for an enterprise allows for the selection of the best investment projecting strategy, which is the process of forming such an enterprise production volume that will be realized with the best guaranteed value of the ratio - return/risk. In the work carried out the practical implementation of the proposed econometric methodology for the selection of the optimal strategy for controlling investment projecting. Using the real investment projecting object as an example, the application features of the proposed methodology are considered, the necessary calculations are performed and the obtained results are analyzed. This methodology can be used as the basis for the development of a modern toolkit for optimizing the control of investment projecting, capable of generating solutions for the practical implementation of specific investment projects. The results of the study can be used by any business entity engaged in investment activities, and, of course, will improve the effectiveness of its performance and competitiveness.

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Longitudinal and Torsional Shock Waves in Anisotropic Elastic Cylinders

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Discontinuities in solutions of a one-dimensional hyperbolic system of equations describing nonlinear longitudinal and torsional waves propagating in elastic rods are studied. The amplitude of the discontinuities is assumed to be small, so that all nonlinear terms, except for the quadratic ones, are neglected in the equations. The form of the shock adiabat and the evolutionary conditions are investigated depending on the parameters of the model. The results of this study are applicable not only to waves in rods, but also to shock waves in anisotropic elastic media.

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Rarefied Gas Between Two Coaxial Cylinders – Transient Heat Transfer: The Effect of Pulsating Radial Motion of the Outer Cylinder. Acoustic Waves Modeling

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On the basis of our previous studies in the flow modeling of rarefied gas between coaxial cylinders under different temperature and kinematic boundary conditions, the energy transfer in the system of gas - surrounding cylinders is studied in the radius pulsating of one of the cylinders. The energy transfer is modeled with continuous model based on Navier-Stokes Fourier equations of motion and energy transfer and with a statistical DSMC model.

The obtained results show that the external mechanical disturbances in the radial direction influence more significantly the operation of the Pirani gauge when there is a difference in the temperature of the thread and the wall of the outer cylinder. These results make it possible to assess the gauge sensitivity.

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Analytic Representation of the Order Parameter Profiles and Susceptibility of a Ginzburg-Landau Type Model with Dirichlet-Dirichlet Boundary Conditions on the Confining the Fluid Walls

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In this work, we study a critical thermodynamic system, say, a simple fluid or a binary liquid mixture, of plane film geometry whose stable states, at given temperature and external ordering field, are determined by the minimizers of the one-dimensional counterpart of the standard ϕ^4 Ginzburg-Landau Hamiltonian in terms of the order parameter. We focus on the case of Dirichlet-Dirichlet boundary conditions on the confining the fluid walls. Assuming that the boundaries of the system are positioned at a finite distance from one another, we solve the corresponding boundary-value problem of one nonlinear differential equation in terms of Weierstrass and Jacobi elliptic functions and give analytic representation of the order parameter profiles and of the local and total susceptibilities depending on the temperature and ordering field.

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Processing Epidemiological Data Using Dynamic Mode Decomposition Method

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Modeling the spread of infectious diseases is extremely challenging due to the lack of a single set of physics-based governing equations. Many current infectious diseases models tend to be based on historic data.

The Dynamic Mode Decomposition (DMD) is an equation-free, data-driven matrix decomposition that can provide accurate reconstructions of spatio-temporal coherent structures arising in nonlinear dynamical systems. The equation-free aspect of operating solely on data snapshots of DMD, can help in the analysis of infectious disease data.

In this work, the Dynamic Mode Decomposition method is applied to malaria infectious disease using historical data from World Health Organization (WHO) and Institute of Health Metrics and Evaluation (IHME), Global Burden of Disease (GBD). Several cases are analysed by using snapshots of infectious disease data concerning malaria death by region, incidence of malaria, and malaria death rates at different space locations. The examples show how DMD can extract the relevant spatio-temporal patterns from the data. Each location's time series is normalized in mean and variance, allowing for a better visual comparison.

A quantitative evaluation of the spatial modes computed from the DMD decomposition and a rigorous error analysis for the reconstruction of data are performed. We emphasize also additional advantages of the reduced order model.

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Hybrid models of ion-acoustic collisionless shock

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Numerical modeling of nonlinear wave processes in media with dispersion, to which plasma primarily relates, plays an important role in studying of a number physical phenomena. In particular, ion-acoustic electrostatic collisionless shocks have been observed both in the high-energy particle fluxes in the Earth's magnetosphere and laboratory experiments with laser plasma. Currently, one of the main questions of cosmology is the mechanism of generation of cosmic rays, which significantly affects processes in near-Earth space. Observational astronomical data indicate that the source of these rays is supernova explosions and ion acceleration at the front of the generated collisionless shock waves. Coulomb collisions are negligible in collisionless shock and cooperative effects of interaction of plasma and electric and magnetic fields play essential roles in the shock formation. The theoretical substantiation of ion-acoustic electrostatic shock waves was created more than half a century ago, but one of the main problems – the acceleration of particles at the shock-wave front – has not yet been solved. The kinetic model and two types of hybrid models of the formation of ion-acoustic shock waves in a collisionless plasma are considered. The regimes accompanied by the overturning of shock waves and the generation of fast particles reflected by the potential barrier of the wave front were studied. We have also investigated the case of three-component plasma with a heavy and light ion.

Acknowledgements. The research was supported by the Russian Foundation for Basic Research 18-29-21025.

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Canonical Deformations of Pseudo-Riemannian Space

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The paper presents the study on the special deformations of a pseudo-Riemannian space which were called canonical. These deformations are characterized by the tensor of metrics deformation is proportional to Ricci tensor. This approach seems to be more appropriate for applications than the standard one, for example, for simulating real physical situation when gravity fields are considered.

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Radiation Effects in Space Silicon Solar Cells: Numerical Models and Software

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Improvements to solar cell efficiency and radiation hardness that are compatible with low cost, high volume manufacturing processes are critical for power generation applications in future long-term NASA and DOD space missions. In this paper, we provide the results of numerical simulation of the radiation effects in a novel, ultra-thin (UT), Si photovoltaic (PV) cell technology that combines enhanced light trapping and absorption due to nanostructured surfaces, separation of photogenerated carriers, and increased carrier density due to UT thickness. Such solar cells have a potential to achieve high conversion efficiencies while shown to be lightweight, flexible, and low-cost, due to the use of Si high volume techniques. Regher Solar is developing manufacturing technology for UT high efficiency silicon solar cells, which can be quickly transferred to mass production and achieve < \$1/W manufacturing cost to supply the booming space economy. To achieve high efficiency on thin wafers Regher Solar is using bifacial amorphous/crystalline silicon heterojunction technology and copper electroplating which can enable ultrathin silicon solar cells with up to 24% AM0 efficiency. Higher efficiency solar cells can reduce solar array mass, volume, and cost for space missions. When solar cells are used in outer space or in Lunar or Marsian environments, they are subject to bombardment by

high-energy particles, which induce a degradation referred to as radiation damage. Radiation tolerance (or hardness) of this UT Si PV technology is not well understood. Research, review, and analysis of solar-cell radiation-effects models in literature have been conducted, and physics-based models have been selected and validated [1]. Several different engineering approaches have been investigated to improve Si solar cell radiation hardness. These include Material/ Impurity/Defect Engineering (MIDE), Device Structure Engineering (DSE), and device operational mode engineering (DOME), which have been shown to be effective in reducing the effects of displacement damage in Si based devices [3]. Lithium-doped, radiation-resistance silicon solar cell is an attractive experimentally proven possibility [2].

In this paper, we provide the results of the accurate numerical simulation of the radiation effects in UT Si PV cells, and radiation damage mitigation techniques. The results of numerical simulation of the radiation effects, coupled with the phenomenon of non-uniform vacancy creation (i.e., maximum displacement damage occurs near the Bragg peak, as described earlier), further indicate that a high-energy protons will cause minimal damage in the ultra-thin 50 μm (or thinner) Si solar cell. These results show that this UT Si PV cell technology can be used for space applications in the high radiation environment.

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Amplification of Longitudinal EM Waves in Graded Low-Epsilon Materials (GLEM)

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Several interesting and practically important new features of EM wave interaction with GLEM related to image processing and optical communications are discussed in [1,2]. We will consider specifics of EM wave interaction with graded semiconductor materials during dynamic holographic transmission grating recording in area near plasma resonance, where dielectric function ϵ depend on electron concentration over plasma frequency. For the monochromatic component of electric field E, wave equation will be [3]:

$$\nabla E = -k_0^2 \epsilon E - \nabla \left(\frac{E \nabla \epsilon}{\epsilon} \right) + \nabla \left(\frac{\rho}{\epsilon} \right)$$

where k_0 is wave vector, ρ denotes free charge density.

In Drude model dielectric function depend on electron density n via plasma frequency. Interference pattern of two intersecting laser beams by photo-excitation spatially modulate electron concentration n, producing diffusion micro-currents, and creating dynamic holographic grating. Recording beams diffract on this dynamic gratings creating feed back with coupling of these two recording beams.

Nonlinear system of partial differential equations describe interaction of high-frequency and low-frequency EM waves with coupling between transversal and longitudinal waves. We suggest solution based on series on dynamic grating thickness that allow to describe effects of transient energy exchange between laser beams and generation of transient self-induced holographic current. Both effect will be increased in GLEM.

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Identification of Holling-Tanner Model with Incomplete Information about Model Parameters

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A new method for numerical evaluation of unknown coefficients for the nonlinear system of differential equations for the Holling-Tanner model is presented. Inverse problem of parameter identification of the model is considered with incomplete information about the model parameters. The equation for the known parameters is linear with respect to five constraints out of thirteen constrains. The method proposed does not use the least square method. The numerical examples illustrate the method.

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Statistical Modeling of CuBr-Ne-HBr Laser Characteristics at High Repetition Frequency

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The investigation of main laser characteristics is an important factor in finding solutions to improve the experimental work in laser technologies. The subject of our study is CuBr-Ne-HBr laser, used as a high-frequency amplifier in active optical systems. This type of systems has been actively developed in recent years and is applied for visual control and diagnostics of fast processes, screened with background illuminations [1]. The aim of the study is to apply statistical methods for the collected experimental data to establish the basic dependencies between the laser characteristics in the conditions of the high repetition frequency of the lasing pulses and the reduced energy deposition in the discharge. Multivariate analysis, such as factor analysis, polynomial type regression of the second degree and more methods are applied. Statistical models have been built that show very good fit with the experimental data. A comparison was made with the results obtained in

[2] by means of numerical methods. By performing different simulations predictions of the values of the output laser power and other laser characteristics are obtained.

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Application of Series with Recurrently Calculated Coefficients for Solving Initial-Boundary Value Problems for Nonlinear Wave Equations

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For one class of nonlinear wave equations with a small parameter, an initial-boundary value problem with zero boundary conditions is considered. The solution of such a problem is constructed in two ways using series with recurrently calculated coefficients. In one case, the method of special series is used, which is based on the choice of some functions (basic functions), by the powers of which the solution of the original problem is constructed as a series with recurrently calculated coefficients. The sequential calculation of the coefficients of the series in this case is based on the special properties of the basic functions. Another way of representing solutions to the problem is based on a combination of Fourier and small parameter methods. It is known that the application of the Fourier method to the representation of solutions of nonlinear partial differential equations leads to an infinite system of ordinary differential equations. For an approximate solution of such a system, a truncation procedure is used. As a result, a nonlinear system of ordinary differential equations remains, defined by the first N harmonics. In the proposed approach, the truncated Fourier series defined by the first N harmonics as the zero member of the series is used, and the small parameter method to find the remaining members of the series is used. The construction of the Lyapunov function for a nonlinear system, which determines the first N coefficients of truncated Fourier series, made it possible to prove that these coefficients are bounded functions for any time and any number N. It is shown that both proposed constructions of series with recurrently calculated coefficients converge to the solution of the initial-boundary value problem on a finite time interval.

Acknowledgements. Russian Foundation supported the work for Basic Research 19-07-00435.

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Hybrid Model of the Open Plasma Trap

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Currently, a new concept of effective plasma confinement in the open traps diamagnetic regime is developing to solve the problem of controlled thermonuclear fusion[1]. In this regime the plasma pressure is close to the magnetic field pressure, the magnetic field in the region occupied by the plasma is close to zero, and in a thin layer at the plasma boundary the diamagnetic "bubble" rapidly increases. The ratio of the maximum plasma pressure to the magnetic field pressure (β) in the tokamak is $\beta = 0.1$, whereas in open traps stable plasma confinement with $\beta = 0.6$ is demonstrated [2]. The usage of the diamagnetic plasma confinement principle in an open trap is of great interest, since it makes possible the creation of a compact thermonuclear reactor. To check the principles underlying the idea of diamagnetic plasma confinement, theoretical studies and mathematical modeling are needed. In this work the hybrid mathematical model of an axisymmetric plasma trap based on the kinetic description for the ion component of the plasma and the MHD approximation for the electron component is presented. Based on the hybrid model, the two-dimensional algorithm for studying the injected particles dynamics in the trap field has been developed. The motion of the ion component is calculated by the particle-in-cell method (PIC), and finite-difference schemes are used to calculate the magnetic field and the electron component of the plasma. On the basis of the developed algorithm, the program code for studying the mechanisms of the self-consistent magnetic field structure forming has been created.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (project 18-31-00314).

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Numerical Study of the Influence of the Two-Burner Heating upon the Heat Transfer during Pyrolysis Process used for End-of-Life Tires Treatment

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End-of-Life tires (EOLT) are ones of the most dangerous waste in the world. They do not practically decompose in nature. Because of this their sound treatment is needed for the environment protection. One of the possible methods for such a treatment is pyrolysis process. It is well known [1,3,4] that globally around 23% of all EOLT are processed through pyrolysis, whereas in the Republic of Bulgaria only 5% are processed by this method. Thus for Bulgaria it becomes clear that the pyrolysis method for EOLT treatment still has a good potential for usage, development and further research. This method is very complicated for modeling and studying, because it is 3D and non-stationary. In our previous works [1,2,5] we created an adequate mathematical model and numerical algorithm in MATLAB for numerical solving the mathematical initial and boundary value problems for the model equations which describe pyrolysis process used for the treatment of the End-of-Life tires (EOLT). Some results for the temperature field for several characteristic periods of operation of pyrolysis station are presented and commented in the paper [1,2,5]. In our next paper [5] we have examined the influence of the heating and the heater position upon the heat transfer during the pyrolysis process used for EOLT treatment.

This paper deals with studying the influence of two-burner heating upon the heat transfer during EOLT treatment by pyrolysis process. The results for temperature fields, temperature isolines and gradients at some specific moments of time and for two different initial heating functions are graphically presented and commented. Results from this modeling can be used in the real pyrolysis stations for more precise displacement of heathers and measurement devices and for designing of automated management of the process.

Keywords: End-of-Life tires, Pyrolysis, Heat transfer modeling, Two-burner heating

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New Nonlinear Parametric Conversion Mechanism for coherent THz Generation

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In the recent studies on coherent THz generation from femtosecond laser pulses in cubic type media the process is explained by optical rectification mechanism which requests strong second harmonics (SH) with energies at least 10-20% of the

main wave. However, such strong SH are not observed in the experiments. The purpose of our investigations is to present new nonlinear parametric conversion $\chi^{(3)}$ mechanism, leading to coherent THz generation. We demonstrate significant increase of THz signal and the possibility for its spectral management.

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Kulish-Sklyanin Type Models: Integrability, Reductions and Soliton Solutions

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This work is a continuation of our previous works [1,2]. We will formulate Riemann-Hilbert Problems (RHP) with canonical normalization compatible with the Mikhailov reduction groups. We show, on the example of $\mathbb{Z}_h$ and $\mathbb{D}_h$-reduction group that the choice of the contour, as well as the choice of the x and t-dependence of the sewing functions $G_k(x,t,\lambda)$ must be compatible with the choice of the simple Lie group $\mathcal{G}$ and the realization of the reduction group [2].

Using Zakharov-Shabat theorem we are able to construct a family of ordinary differential operators for which the solution of the RHP is a common fundamental analytic solution. Thus we are able to construct new types of integrable nonlinear evolution equations and construct their soliton solutions.

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Two-Step Time Series Analysis for Air Pollution in Relation to Weather Conditions: Case Study of Nessebar, Bulgaria

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Air pollution is a major problem in many urban areas in Bulgaria, harmful to the human health. In this study, based on a large number of observations for particulate matter 10 micrometers or less in diameter (PM10) and concomitant meteorological conditions, the task of mathematical modeling of the time series and predicting the level of future concentrations is set. As a case study, data about the town of Nessebar, a typical seaside city, were used. The collected data are daily averaged for the period February 2015 – March 2018. Using the autoregression moving average (ARIMA) method, models of the considered time series are built. To obtain more realistic forecasts, the methodology is implemented in two steps. The first step is to build univariate ARIMA models for any of the meteorological variables and to predict their future values. In the second step, the calculated forecasted values are applied to construct ARIMA PM10 models and to estimate the forecasts of this pollutant for a short time of 3 days ahead. The obtained models agree well to the known observed values. The proposed approach can be applied to other type of pollutants. It does not depend on additional forecasts from other sources and allows the development of a software application to predict future levels of pollution depending on the meteorological hazards.

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On the Derivative Nonlinear Schrödinger Equation Related to Symmetric Spaces

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We will present multi-component generalizations of derivative nonlinear Schrödinger (DNLS) type of equations having quadratic bundle Lax pairs related to $\mathbb{Z}_2$-graded Lie algebras and **A.III** symmetric spaces. The Jost solutions and the minimal set of scattering data for the case of local and nonlocal reductions are constructed. Furthermore, the fundamental analytic solutions are constructed and the spectral properties of the associated Lax operators are briefly discussed. The

Riemann-Hilbert problem for the multi-component generalizations of DNLS equation of Kaup-Newell and Gerdjikov-Ivanov types is derived. A modification of the dressing method is presented allowing the explicit derivation of the soliton solutions for the multi-component GI equation with both local and nonlocal reductions. The infinite set of integrals of motion for these models is briefly described at the end.

Based on [1] – a joint work with Vladimir Gerdjikov and Rossen Ivanov.

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Mappings of Special Quasi-Einstein Spaces

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The paper treats conformal and geodesic mappings of quasi-Einstein spaces of a special type. We found the condition of closedness for these spaces in relation to other mappings. Quasi-Einstein spaces include Einstein spaces as a sub-class, so presented results strengthen the results obtained previously for Einstein spaces.

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An Integrated Software Package for Estimating Parameters of the Brown Bear (Ursusarctos L.) Population

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One of the best habitats of brown bears (Ursusarctos) which are a strict protected species in Europe is located in Bulgaria. Monitoring populations of protected wildlife species is necessary for effective management and conservation of their habitats. In this work, we present the program tool named ArctosPop for automatic estimation of the brown bear (Ursusarctos L.) population size in Bulgaria. The computing programme integrates statistical algorithms, which use as input data the observed data for traces of brown bears during National monitorings. The main features of the software tool are presented and guidelines for improvement of the programme.

Keywords: Statistical estimation, Data analysis, Brown bear population

Acknowledgements. This work was supported by the Bulgarian Ministry of Education and Science under the National Research Programme "Environmental Protection and Reduction of Risks of Adverse Events and Natural Disasters," approved by the RCM #577/17.08.2018 and signed Agreement DO-#230/06.12.2018.

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A Short-Term Interest Rate Merton's Model Influenced by a Risk Market Factor

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In the context of the interest rate derivatives, a short rate model is a mathematical model that can predict the random movement of the interest rates. During the last century different types of rate interest models are derived. Examples for this are the works [1,2,3,4] and many others. In the present paper we introduce a short-term interest rate Merton's model for which the movement of the interest rate is given by a stochastic differential equation. For this model we consider the zero coupon bond's price which is determined by using the apparatus of the stochastic differential equations and the partial differential equations.We use the diffusion equation to calculate the bond's price for this model in the case of risk market factor and without a risk market factor. For determing the zero coupon bond's price we give some numerical experiments and graphics. Some of the experiments are obtained by using the Monte Carlo method.

Acknowledgements. This work has been accomplished with the financial support of the MES by the Grant No. D01-221/03.12.2018 for NCHDC – part of the Bulgarian National Roadmap on RIs and by the financial funds allocated to the St. Kl.Ohridski University of Sofia, grand No 80-10-27/2019.

The second author (M.L.) is partially supported by a project "Stochastic and simulation models in the field of medicine, social sciences and dynamic systems" funded by the National Science Fund of Ministry of Education and Science of Bulgaria (Contract No. DN 12/11/20 Dec. 2017).

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External Estimates and Comparison Principle for Trajectory Tubes of Nonlinear Control Systems

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We consider nonlinear systems with uncertainty in dynamics due to controls or disturbances. Uncertainty generates a whole family of trajectories, called a trajectory tube. By an external estimate of a trajectory tube of the system we mean a multi-valued mapping which contains all trajectories of this family as its selectors. There are various approaches to the construction of external estimates for trajectory tubes. Some algorithms for the construction of estimates are based on discrete (pixel) approximations and approximations by ellipsoids and polyhedra. One common approach is to use Lyapunov type functions. It relies on the study of their behavior along trajectories and uses the comparison principle for differential inequalities. In this talk, we present some modification of this approach associated with the use of a new form of comparison equations. Also, the Lyapunov type functions and the comparison principle are applied to the estimation of the dynamics of systems with integral constraints on uncertain parameters (controls). The results are illustrated with several examples of numerical simulations.

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On Convexity of Small-time Reachable Sets of Nonlinear Control Systems

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The convexity of reachable sets plays an important role in the development of algorithms for solving optimal control problems and problems of feedback control. For nonlinear control systems the reachable sets are generally not convex and may have a rather complicated structure. However, for systems with integral quadratic constraints on the control B. Polyak have shown that the reachable sets are convex if the linearization of the system is controllable and control inputs are restricted from above in L_2 norm by a sufficiently small number. In the present talk we use this result to prove sufficient conditions for the convexity of reachable sets of a nonlinear control-affine system on small time intervals, assuming that control resources are limited by a given (not necessarily small) value. These conditions are based on the asymptotics for the minimal eigenvalue of the controllability Gramian of system linearization as a function of the length of the time interval. We prove the asymptotics for a linear time-invariant system containing a small parameter that implies the convexity of small-time reachable sets for some classes of two-dimensional nonlinear control systems. The results of numerical simulations for illustrative examples are discussed.

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Solution of the Problems of Nonhomogeneous Incompressible Fluid Dynamics by the CABARET Method

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Method for solving a nonhomogeneous viscous incompressible fluid dynamics is proposed using CABARET scheme. We study the problem of spot dynamics in a fluid that is stably stratified by density. The comparison with the results of other works and with analytic solution is considered. The statement of the problem was obtained from the consideration of a spot with mixed salt water placed in a solution with the steady-state distributions of the salinity and density fields. We will assume that the spot is placed in a rectangular tank with sufficiently remote liquid-tight walls. The spot has the form of a cylinder. Density stratification is given by a linear function of height, and buoyancy forces are modeled as a deviation of density from a stable stratification. The problem is solved in a two-dimensional formulation with different regimes determined by Reynolds and Froude numbers. There are no restrictions on the smallness of the density deviations therefore theoretically this technique can be applied for problems with a complex dependence of density on height. For example, in the problem of the occurrence of internal waves in a stratified atmospheric layer, where, as a result of solar radiation, the heated at the surface air forms ascending currents.

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Tiny Giants – Mathematics Looks at Zooplankton

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Zooplankton is an immensely numerous and diverse group of organisms occupying every corner of the oceans, seas and freshwater bodies on earth. They form a crucial link between autotrophic phytoplankton and higher trophic levels such as crustaceans, mollusks, fish, and marine mammals. Changing environmental conditions such as rising water temperatures, salinities and decreasing pH values currently create monumental challenges to their well-being.

A significant subgroup of zooplankton are crustaceans of sizes between 1 and 10mm. Despite their small size they have extremely acute senses that allow them to navigate their surroundings, escape predators, find food and mate. In a series of joint works with Rudi Strickler (Department of Biological Sciences, University of Wisconsin-Milwaukee) we have investigated various behaviors of crustacean zooplankton. These include the visualization of the feeding current of the copepod Leptodiaptomus sicilis, the introduction of the "ecological temperature" as a descriptor of the swimming behavior of water fleas Daphnia pulicaria and the communication by sex pheromones in copepods. The work draws from optics, ecology, neuroanatomy, computational fluid dynamics, and computational neuroscience.

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New Trend Criteria for Monitoring the Conditions of Technical Objects

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Various trend criteria are widely used in technical diagnostic systems (they are the part of technical condition monitoring algorithms). The trend criteria allow to establish the fact of emergence and development of the trend in data time series recorded during the diagnostic objects operation. The presence of time series trend component can be the result that monitored parameter comes out from acceptable limits and the result of an occurred fault. The proposed well-known trend criteria are scalar, since the time series counts of one registration parameter are the argument of trend statistics. The scalar nature of the trend criteria limits the capabilities of diagnostic systems, since the technical condition of modern object is characterized by multidimensional set of time-varying parameters. The algorithms of trend control, used in modern technical diagnostic systems, allow to reveal the fact only there is a trend at a given level of statistical significance, but the nature of the trend remains unknown. To solve the problems of increasing the reliability of diagnostic conclusions about the conditions of technical objects, an approach to assessing the mutual dynamics of multidimensional trends is proposed. The approach is based on the formation of multidimensional arrays from time series of registration data of an object technical condition – diagnostic parallelepipeds. Analysis of the diagnostic parallelepipeds is performed by decomposing them by singular numbers. In order to identify mutual trends and trends of differences, it is proposed to choose two time series and their counts with the same arguments should be combined into one complex number. The constructed time series is decomposed into components using the principal component method. The proposed approach allows to perform the selection of an object condition parameters on groups that have trends of the same type or do not have ones. As a result, it becomes possible to localize faults and to increase the reliability of diagnostic conclusions about the object technical condition.

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Analysis of a Predator-Prey Model with SEIRS Epidemic in the Prey Population

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In this paper, the author studies a predator-prey model based on epidemic disease. The epidemic disease is described by SEIRS (Susceptible – Exposed-Infectious – Recovered – Susceptible) model with logistic growth function in the susceptible prey population. The fifth dimensional eco-epidemiological model consist five classes: susceptible prey $S(t)$, exposed prey $E(t)$, infected prey $I(t)$, recovered prey $R(t)$ and predator $P(t)$. The function of response is supposed to be of Lotka-Volterra type. The dynamics of the system has been studied by proving conditions of existence and the stability of the equilibrium points and the solution is bounded. Since the model is based on the SEIRS epidemiological model, the basic reproduction number R_0 is calculated. The infection will die out in the long run when $R_0 < 1$ otherwise there is an epidemic in the prey population. The presented numerical results are consistent with the theoretical ones and show the dynamical behaviors of the described model.

Acknowledgements. This work was supported by the National Scientific Programme "Information and Communication Technologies for a Single Digital Market in Science, Education and Security (ICTinSES)," approved by the RCM #577/17.08.2018 and financed by the Bulgarian Ministry of Education and Science.

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A Novel 3D Visualization Approach: a Proof-of-Concept Study on the Histidine Residues in Myoglobin

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Geometrically, proteins can be viewed as discrete piecewise linear curves that allows for their description in terms of extrinsic spatial variables, such as the Ramachandran angles. However, they are also characterized by additional two — intrinsic and independent — geometric structures determined by the peptide planes and the side chains. We present a *proof-of-concept* study towards the development of a novel 3D visualization method, the *framing 3D visualization*, based on the construction of a series of orthonormal coordinate frames along the protein side chains and mapping the atoms positions onto a unit sphere. We analyze distal and proximal histidine residues in myoglobin. The results are in good agreement with biological data and provide a new perspective for further understanding of structure and function of histidines in myoglobin. This suggests the method can reliably depict the spatial orientation of side-chain covalent bonds in a protein and may eventually be advanced into a full-scale visual tool for protein-structure analysis, complementary to the currently used visualization suits.

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Hamiltonian Approach to Nonlinear Water Waves – The Long Wave Approximation

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We derive model equations for water waves propagating over uneven bottom using the Hamiltonian formulation of the water-wave problem. The assumptions are for a homogeneous incompressible, inviscid, fluid medium bounded by a free surface. The Hamiltonian of the system is expressed in terms of the so-called Dirichlet–Neumann operators. Specific scaling of the variables is selected which leads to approximations of Boussinesq and Korteweg–de Vries (KdV) types, taking into account the effect of the slowly varying bottom. Underlying current could be taken into account as well. The arising KdV equation with variable coefficients is studied numerically.

Joint work with A. Compelli, C. I. Martin and M. Todorov

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Probabilities for p-Outliers – General Properties

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The task for a general and useful classification of the heaviness of the tails of probability distributions still has no satisfactory solution. Due to lack of information outside the range of the data the tails of the distribution should be described via many characteristics. Index of regular variation is a good characteristic, but it puts too many distributions with very different tail behavior in one and the same class. One can consider for example Stable(α) and Hillhorror(α) with one and the same fixed parameter $\alpha > 0$. When analyzing the heaviness of the tail of the observed distribution we need some characteristic which does not depend on the moments because in the most important cases of the heavy-tailed distributions theoretical moments do not exist and the corresponding empirical moments fluctuate too much. In this paper, we show that probabilities for different types of outliers can be very appropriate characteristics of the heaviness of the tails of the observed distribution. They do not depend on increasing linear transformations and do not need the existence of the moments. The idea origins from Tukey's box plots, and allows us to obtain one and the same characteristic of the heaviness of the tail of the observed distribution within the whole distributional type with respect to all increasing linear transformations. These characteristics answer the question: At what extent we can observe "unexpected" values?

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Logarithm of Ratios of Two Order Statistics and Regularly Varying Tails

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Here we suppose that the observed random variable has cumulative distribution function F with regularly varying tail, i.e. $1 - F \in RV_{-\alpha}$, $\alpha > 0$. Using the results about exponential order statistics we investigate logarithms of ratios of two order statistics of a sample of independent observations on Pareto distributed random variable with parameter α. Short explicit formulae for its mean and variance are obtained. Then we transform this function in such a way that to obtain unbiased, asymptotically efficient, and asymptotically normal estimator for α. Finally we simulate Pareto, Fréchet and Log-logistic samples and show that in any of these three cases this estimator gives good results. We consider also Hill-horror cases because although this distribution has RV tail it turns out that our estimator is not appropriate in that case. The paper finishes with a simulation study which depicts the benefits of the considered estimator over Hill, t-Hill, Pickands and Deckers-Einmahl-de Haan estimators.

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Models of Mechanical Systems Preserving the Weyl TensorR

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The paper presents conditions sufficient for preservation of Weyl tensor in the course of mapping. The obtained results are applied for the study on the model of dynamic system with some outer forces. The equations are found for the vector of outer forces of dynamic system in the case when prototype and image have the corresponding Weyl tensors. These conditions are differential equations in covariant derivatives from connection of specially constructed space of affine connectivity.

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The Application of Special Hermite Finite Elements Coupled with Collocation to the 3D Poisson Equation

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We propose an efficient method for a reaction-diffusion type equation. To construct a discrete problem, we use special bicubic Hermite finite elements in combination with the collocation technique. This enables one to reduce the dimension of the system of linear algebraic equations. We show that the matrix of the reduced system coincides with the matrix of a system of the finite element method for elements with a smaller number of degrees of freedom. The only difference between two systems is in the right-hand side. Now we can immediately calculate the entries of the reduced matrix without elementary row operations with the original matrix. This significantly simplifies the numerical implementation. However, the order of convergence of an approximate solution is retained, namely, we have fourth-order convergence in the L_2-norm. In numerical experiments the method is applied to the three-dimensional Poisson equation. Numerical results confirm the theoretical estimate.

Acknowledgements. This research is supported by Project 17-01-00270-a of Russian Foundation for Basic Research.

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Real-Time Predicting the Thermal State of the Electronic Unit by Analyzing Trends in the Built-In Sensors Readings

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We propose a method for condition monitoring of an electronic unit (EU) to real-time prediction of thermal state when operation mode changes. The method is based on the analyzing trends in the readings of the built-in temperature sensors. Data for analysis obtained from a laboratory test bench. The EU heat simulator is a rectangular aluminum frame, on which printed circuit board with high-power transistors is installed to simulate the operation of real electronic components. Heat from the transistors is removed from the EU through the flat bottom surface of the frame. The test bench is used to investigate thermal operating modes of the EUs both in the atmosphere and in a vacuum. The test bench is a vacuum chamber with a heat removing base that maintains the temperature, and with a temperature sensing system at control points of the EUs and environment. We assume, there are several operating modes of EU that correspond to a certain set of the dissipating heat active transistors. Thus, there is a fixed set of thermal operating modes, the deviation from which can be interpreted as a malfunction. Earlier it was shown that for the stationary mode a classifier can be constructed to determine the EU operation mode using temperature data. The present research proposes a method for determining the EU operation mode and the final temperatures expected to be reached by analyzing trends in temperature data before reaching a steady state. The change in temperature at a point on the EU surface is approximately described by a linear combination of a small number of exponential functions. Moreover, the exponent in the exponentials is independent of the point and characterizes the EU. On the contrary, the coefficients in the linear combination depend on a point. Thus, the dynamics of the EU temperatures after change operating mode is determined by the exponents and by the coefficients of the linear combination. These parameters can be obtained with using several consecutive temperature measurements. The constant term in the expression predicts the final value of temperature at a given point after stationary state will be reached. Then, using our classifier and the set of the predicted temperatures in every control point, we may predict this operating mode or suspect a malfunction before reaching thermal steady state.

Acknowledgements. This research is supported by Project 18-47-242005 of Russian Foundation for Basic Research.

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Theoretical Model of Elastic Disc Lifting from a Water Surface

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A new model of the initial stage of lifting of a disc from a water surface was developed. Experiments were performed in Fluid Mechanics Group, Universidad Carlos III de Madrid to validate the linearized model of water exit by Korobkin [1]. In these experiments, an acrylic disc was pulled vertically from a water surface at a large acceleration. Two phenomena have been observed in the experiments, which could not be explained by the theory. First, the acceleration of the plate does not monotonically increase in time even if the external force does. Second, the wetted part of the plate does not start instantaneously to decrease with the plate lifting. Instead, there is an initial interval of time during which the wetted area of the plate does not shrink. It was assumed that both these phenomena are caused by the elastic behaviour of the plate, even though these vibrations were not visible in experiments. Guided by this assumption a new model of water exit of an elastic body was developed. This model is based on a linearized model of water exit and it describes unsteady axisymmetric flow generated by a lifted elastic body coupled with its elastic deformations. The disc displacement was presented as superposition of a rigid body motion and a series of normal elastic modes of a free-free circular disc supported at its center. Some additional mass, which corresponds to the experimental equipment, was included in the model. It should be emphasized that this theoretical model has no free parameters to adjust. The elastic parameters of the disc, mass of the equipment and the force evolution measured in the experiments are used to determine the plate deflection, acceleration and hydrodynamic pressure. The theoretical results obtained with only one mode agree very well with experimental data and explain both of those non-intuitive phenomena, mentioned before. The theoretical model was additionally validated by the experiments with circular discs of different rigidities [2].

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Stability of the Pexiderized Trigonomertric Functional Equation

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We investigate the superstability of the Pexiderized trigonomertric functional equation.

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On Task of Thick-Walled Aluminum Pipe under High Temperature Deformation

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The objective of the presented research is the mathematical formulation and analytical solution of the task on thick-walled pipe loading by the external pressure and by the stretching force in the thermal range of dynamic superplasticity. Used the definition of "dynamic superplasticity" emphasizes the hierarchy of states, which occurs in the material from the source various-grained structure in a changing temperature and rate conditions. The utilization of the dynamic superplasticity effect is one of the most perspective technological techniques of metal processing. The purpose of such processes is creation of the resource-saving metal forming processes to produce metal billets and parts with predetermined grain structure and mechanical properties. The mathematical formulation of the problem includes differential equations of equilibrium; kinematic relations, establishing connection between strain rates and displacements; the condition of incompressibility in strain rates; the defining ratios in the form of the equations of the theory of elasto-plastic processes of small curvature; the state equation in the form of the dependence of stress intensity on strain rate intensity and which is a consequence of the dynamic model associated to isothermal process. The state equation is suitable for the description of regularities of high-temperature deformation in the wide strain rate-rate interval including the conditions of dynamic superplasticity effect realization. Joint consideration of these equations leads to definition of one allowing function which finding allows to establish stress fields, strain rates and displacements rates.

Comparison of the obtained theoretical and available experimental data was carried out for the alloy $AlMg_5$. The problem is the basis for the possible development of theories of technological processes such as compression and distribution of pipes, drawing and autofreting in superplasticity.

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Mathematical Modeling of Bone Fracture Repair

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Several new mathematical models are presented that describe the process of bone fracture repair. The models incorporate the complex interactions between immune cells and bone cells at the fracture site. The resulting systems of nonlinear ordinary di?erential equations are studied analytically and numerically. Mathematical conditions for a successful bone fracture repair are formulated. The models are used to numerically monitor the evolution of a broken bone for different types of fractures and to explore possible treatments that can accelerate the bone healing process

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Solitary Waves to Boussinesq Equation with Linear Restoring Force

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The Boussinesq equation with linear restoring force

$$U_{tt} - U_{ttxx} - U_{xx} + U_{xxxx} + mU = (\alpha u^p)_{xx}, \quad (t, x) \in \mathbb{R}^+ \times \mathbb{R},\ m > 0,\ p \geq 2$$

models the transverse deflections of an elastic rod on an elastic foundation. In this talk we investigate solitary waves to this equation. Depending on the velocity of the wave and the restoring force coefficient m we obtain explicit expressions for the asymptotic behavior of the solitary waves. To compute the solitary waves we apply the spectral method proposed by Petviashvili. We test the accuracy of the computations on nested grids. We perform several simulations to analyze the impact of the restoring force coefficient m and the parameters of nonlinearity on the shape and the support of the solitary waves.

Acknowledgements. This work is partially supported by the Bulgarian National Science Fund under Project H 22/2 from 2018.

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On the Stability of a Steady Convective Flow Due to Nonlinear Heat Sources in a Magnetic Field

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Consider a layer of a viscous incompressible fluid bounded by two vertical planes. There exists a steady flow in the vertical layer caused by internal heat generation. The heat sources are distributed within the fluid in accordance with the Arrhenius' law. A magnetic field of constant strength is applied in the direction perpendicular to the planes. The flow is characterized by four dimensionless parameters: the Grashof number, the Prandtl number, the Hartmann number and the Frank-Kamenetsky parameter. This problem is important in applications such as biomass thermal conversion. The objective of the study is to determine the factors that enhance mixing and lead to more efficient energy conversion. The problem is described by a system of magnetohydrodynamic equations under the Boussinesq approximation. The nonlinear system of ordinary differential equations describing the steady flow is solved numerically. Linear stability of the steady flow is investigated using the method of normal modes. The corresponding linear stability problem is solved numerically by means of a collocation method. The solution is found for different values of the parameters characterizing the problem. Two destabilizing factors are identified from the numerical solution of the stability problem: the increase of both the Prandtl number and Frank-Kamenetsky parameter destabilize the flow. On the other hand, the increase of the Hartmann number stabilizes the flow. It is also shown that marginal stability curves for high Prandtl numbers can have a cusp or even closed loops where the steady flow is unstable with respect to two normal modes.

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Numerical Methods for Mean Field Games with Discontinuous Control Function

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In this study, the numerical methods are proposed for non cooperative Mean-Field Games with a continuum of players. These games frequently arise in the economic theories, production of exhaustible resources, environmental policy, and other population models. Firstly, Mean Field approach was proposed by P.-L. Lions and J. M. Lasry and was inspired by ideas in statistical physics in which the individual particles-players are considered in terms of mean field. Mean Field statements lead to forward-backward structure of the equilibrium given by the coupled system of two parabolic partial differential equations: the Fokker-Plank-Kolmogorov equation and the Hamilton Jacobi-Bellman one. This study focuses on the discrete approximation of these equations similar to the application of the MFG theory directly at discrete level. A part of Mean Field Game problems is based on statements in which the control functions are not continuous (e.g. price competition models, congestion models). Here we present the numerical algorithm for solution of such problems with the difference schemes which are based on the semi-Lagrangian approximations and improve properties of discrete problems.

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Step-By-Step Nanostructures Formation under Uniform Laser Field

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In recent decades, nanostructures with unique properties, which may differ from properties of a bulk sample, and dependent on both composition and shape have been studying actively. Therefore, the problem of developing a universal method of nanostructures formation is a subject of interest of many scientists. One of the least expensive approach is based on the processes of nanoparticles self-assembly. In this study, a dynamic self-assembly model for a triple of particles under laser field using Brownian dynamics is proposed. The possibility of forming a three-particle structure with a predetermined geometry is studied in variant of step-by-step formation: the first one is formation from a preformed pair of particles fixed in space; another one – from a preformed pair of particles not fixed in a space. The geometry of resulting nanostructures is shown to be determined by the polarization direction of laser radiation and the laser wavelength. Under proper choice of these parameters the formation of structures is shown to be highly efficient. It was shown that maximum probability of structures formation can reach 36–46% per single laser pulse of 10ns duration. The obtained results allows to make significant progress in the study of the method of structures self-assembly in the field of laser radiation, which can be use as universal method to form structures with specified properties that will find application as sensors, photodiode elements and solar cells.

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Impact onto Floating Ice

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The unsteady two-dimensional problems of a rigid body impact onto a floating elastic plate are studied. Both the plate and the water are at rest before impact. The thickness of the plate is constant. The motion of a short plate is caused by the impact force transmitted to the plate through an elastic layer with viscous damping on the top of the plate. Elasticity of a short floating plate can be neglected in the leading order, see [1]. The hydrodynamic force acting on the short plate is calculated by using the second-order model of plate impact in [2]. The present study is concerned with the deceleration experienced by a rigid body during its collision with a floating object, see [3]. The presence of the elastic layer between the impacting bodies may lead to multiple bouncing of them, if the bodies are relatively light, before their interaction is settled and they continue to penetrate together into the water. In the case of long ice sheets, vertical impact on ice generates shear and compression waves in the ice plate, which may result in fracture of the plate. The linear theory of elasticity is used to find the maximum stresses in the ice and their locations. This study is motivated by ship slamming in icy waters, and by the effect of ice conditions on conventional free-fall lifeboats.

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Matrices Diagonalization in Solution of Partial Differential Equation of the First Order

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A new approach to solving systems of linear partial differential equations of the first order has been offered. We use the methods of simultaneous reduction of several matrices. Sometimes, this allows to get an analytical solution or significantly simplify the problem. Systems of linear partial differential equations of the first order are used in various application areas. Autonomous equations are described with several constant matrixes of coefficients. In this paper, we consider the case when there are over two matrices. A case of all matrixes being singular is considered possible. Nondegenerate linear transformations of the system are the replacement of variables and the multiplication of the system on the left by a nonsingular matrix.Thus, the transformation of initial matrixes comes down to simultaneous multiplication of them by a matrix on the left and by another matrix on the right. When one of the matrixes is nonsingular, the system is multiplied on the left by the matrix which is inverse matrix to this matrix, and then we use the similarity transform of the other matrices. We find the transformation using the method of the commutative matrix. It consists in finding in the set of all matrices commuting with these matrices such a matrix $\mathbf{T}$, which has at least two different eigenvalues. The vectors of the canonical basis of the matrix $\mathbf{T}$ are the columns of the desired similarity transformation matrix. Such a similarity transformation leads all the original matrices to the same block-diagonal form with two (at least) blocks on the main diagonal. To find the set of all matrices that commute with original matrices, you can declare all the elements of the matrix as unknowns and make up the corresponding system of linear homogeneous algebraic equations. There are methods for finding a general solution to such a system of equations. If the dimension of the obtained general solution is greater than 1, then the splitting of the original system of equations is possible, otherwise, it is not. If the system can be split into independent equations, then it is easy to find a general solution of the initial system of equations. But even with a partial splitting, the task can be considerably simplified. If all matrixes are singular or are ill-conditioned, then instead of a set of matrixes that are commutable with the given matrixes, pairs of matrixes $\mathbf{T}_1$ and $\mathbf{T}_2$ should be found. Such pairs are solutions to the system of matrix equations $\mathbf{T}_1\mathbf{B}_i = \mathbf{B}_i\mathbf{T}_2$, $i = 1, 2, ..., g$. Here, $\mathbf{B}_i$ are initial matrixes, g is a number of them. A left and right transformation matrix are matrixes which columns are vectors of canonical basis of matrixes $\mathbf{T}_1$ and $\mathbf{T}_2$ correspondingly. As a result, the initial system of equations is split to sub-systems with the help of matrix methods. This simplifies further solution process.

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On a Polyhedral Method of Solving an Evasion Problem for Linear Dynamical Systems

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We deal with a linear conflict control problem with two controls, where the aim of the first one is to steer the trajectory into a given target set at a given terminal time, whereas the aim of the second control (disturbance) is opposite. Thus we have two subproblems, namely the approach problem and the evasion problem. There are known approaches to solving both subproblems for differential systems based on construction of solvability tubes (in other terms, maximal stable bridges, Krasovskii's bridges, backward reachable tubes) and there are known tight interconnections between the solvability tubes and the Pontryagin alternated integral, Hamilton-Jacobi-Bellman equations, and funnel equations. Since practical construction of the trajectory tubes is usually a very complicated problem, various numerical methods have been developed. In particular, constructive computative schemes for the approach problem using an ellipsoidal technique were proposed by A.B. Kurzhanski and then expanded to a polyhedral technique by the author. Their main advantage is that such techniques allow to find solutions by rather simple means. Here we propose a polyhedral method for solving the evasion problem using polyhedral (parallelelotope-valued) tubes. We assume that the terminal set is a nondegenerate parallelepiped and both controls are subjected to given parallelepiped-valued constraints. Ordinary differential vector and matrix equations, which describe the mentioned tubes, are presented. Control strategies, which can be calculated by explicit formulas on the base of these tubes, are proposed. Note that these tubes coincide with external polyhedral estimates for solvability tubes in the approach problem. Also a similar polyhedral method for solving the evasion problem for discrete-time systems is considered.

Acknowledgements. The research was partially supported by the Russian Foundation for Basic Research (RFBR Project No. 18-01-00544a).

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Longitudinal Radiation Forces and Trapping of Particles in Femtosecond Pulses

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As it was demonstrated in [1], it is possible to trap particles by lasers, working in CW regime. The analytical expression of the radiation force is obtained in dipole approximation of the ponder-motor force and is proportional to the transverse gradient of square of the electrical field. The question what kinds of radiation forces exist for laser pulses is still open. In this work we will present analytical expression for longitudinal radiation force of a laser pulse propagating in dielectric media obtained in amplitude approximation. This force is proportional to the second derivative of pulse time envelope. In the femtosecond region this leads to trapping of particles in the pulse.

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Econometric Modeling and Analysis of Brewing Industry Data

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The study of dependencies in multi-dimensional data from the brewing industry is of great importance for increasing the production and realization of the final output for each brand. Numerous factors influence market processes and, in particular, the effectiveness of advertising activities. This work explores the interaction between the main factors – digital and non-digital advertising, atmospheric temperature, price of manufactured products, distribution and others on the sales value growth. Monthly observations over four years have been used for a big brewery brand in Bulgaria. Via the multivariate statistical methods, econometric models have been constructed and applied that allow a description of the data studied and prognosis of the dependent variable for various advertising strategies.

Study of Internal wave in stratified fluid is important, for example, associated with transport and mixing process in the ocean. In particular stable solitary wave with large amplitude and long wavelength is studied intensively in order to clarify the dynamics of its generation and propagation. In theoretical studies, some weakly nonlinear equations which can be systematically derived from fundamental equations have been proposed. These equations, including Korteweg-de Vries (KdV) equation, have the solitary wave solution which interact without deformation. But most pioneer studies were limited to one-dimensional modeling. Concerning the two-dimensional interaction in which line solitons interact with different angle of propagation direction, different kind of interaction can be seen. Depending on the interaction angle the new soliton is generated and steady propagates (soliton resonance). This phenomena was firstly discussed by Miles in 1970 for surface water wave and has studied by many researchers. Recently mathematical studies about integrable Kadomtesev-Petviashvili equation, which is two-dimensional extension of KdV equation, advanced its understanding. In this talk this resonant phenomena is numerically studied by two-dimensional model equations which has one-dimensional line soliton solution. It is clarified that the "nonlinear resonant" phenomena can be seen not only the integrable systems but also the non-integrable model equation.

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Optimization of Fire Station Locations through Genetic Algorithms

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Determination of where to locate fire stations and how many fire stations to have in a given area is perhaps the most important decision faced by any fire authority in the UK. Fire station facility location problems have multiple objectives and are complex and NP-hard. Multiple objectives often conflict with each other and require multi-objective approaches rather than a single objective approach.

Our research aims to determine the optimal location of fire station facilities. Various fire risk categories of the given area are considered in the location optimization in order to establish a fire station location model which is understandable and practical to fire service authorities. It has been proposed that fuzzy multi-objective programming is combined with genetic algorithms so as to the original fuzzy multiple objectives can be appropriately converted to an unified 'min-max' goal, which makes it easy to apply a genetic algorithm for the problem solving. The coordinates of N possible fire station locations represent a chromosome in the genetic algorithm and the constraints about the optimal number of fire stations are automatically satisfied in the genetic algorithm.

Our proposed approach has three distinguish features: (a) distinguishing the areas with different risk categories in the optimal location problem is more reasonable and understandable; (b) introducing the fuzzy nature of the recommendations of the Home Office in the UK on the speed of fire engine attack to accidents in the optimal location model has greatly improved the precision of the optimal model and possesses a potential to reduce the amount of facilities; (c) choosing a suitable chromosome format and embedding constraints into the fitness function of a genetic algorithm has dramatically reduced the complexity of the optimal fire station location problem.

The case study was based on the data collected from the Derbyshire fire and rescue service and used to illustrate the application of the method for the optimization of fire station locations. Our case study illustrates that the model established and the method proposed to deal with the constraints are applicable in fire and rescue services.

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Computational Study of Novel Natural Bioactive Peptides in Solution

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Antimicrobial peptides (AMPs) are essential components of innate immunity in most multicellular organisms. Even though bacteria have been exposed to AMPs for millions of years during their co-evolution, they have yet to develop wide-spread resistance to AMPs. This makes them promising alternatives to conventional antibiotics.

Here we present our study on two peptides, isolated from the mucus of Cornu aspersum snails. These peptides are secreted in the form of multipeptide mixtures, that are effective against viruses, bacteria, fungi and cancer cells. We show, that the peptides tend to aggregate and form clusters both in monocomponent and in mixed solutions. In the cluster individual monomers begin to form secondary structure elements between different peptides. The number of hydrogen bonds increases with the number of monomers, participating in the cluster, thus stabilizing the secondary structure elements. These results suggest that the peptides first form mixed clusters which are capable of binding electrostatically to the bacterial membrane and transporting in it hydrophobic neutral peptides.

Acknowledgements. This work was supported in part by the Bulgarian Ministry of Education and Science (Grant D01-217/30.11.2018) under the National Research Programme "Innovative Low-Toxic Bioactive Systems for Precision Medicine (BioActiveMed)" approved by DCM # 658/14.09.2018 and by the Bulgarian Science Fund (Grant KP-06-OPR 03-10/2018). Computational resources were provided by the HPC Cluster at the Faculty of Physics, St. Kl. Ohridski University of Sofia.

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Performance Analysis of Hybrid Parallel Solver for 3D Stokes Equation on Intel Xeon Computer System

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In our previous work we have got studied the performance of a parallel program, based on a direction splitting approach, to solve time dependent Stokes equation. We used a rectangular uniform mesh, combined with a central difference scheme for the second derivatives. Hence, the program needed a solver of tridiagonal linear systems. We were targeting massively parallel computers, as well as clusters of multi-core nodes. We developed an implementation of the proposed algorithm using hybrid parallelization based on the MPI and OpenMP standards. It was motivated by the need to maximize the parallel efficiency of our implementation. Essential enhancements of the parallel code were achieved by introducing two levels of parallelism: (i) between-node level supported by the MPI and (ii) inside-node parallelism supported by the OpenMP.

This paper presents an experimental performance study of the developed parallel implementation on a supercomputer using Intel Xeon processors as well as Intel Xeon Phi coprocessors. The experimental results show an essential improvement when running experiments for a variety of problem sizes and number of cores/threads.

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Temerature Field Distribution in Spoke-Type Permanent Magnet Synchronous Machines

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This paper presents a mathematical model of temperature field distribution in spoke-type permanent magnet synchronous machines. Recently, these machines are actively used for hybrid electric vehicle traction drive application due to their unique merits.

Compared to other studies of thermal processes in a synchronous machine, the originality of this study is to build an integrated thermal model that takes into account the nature of thermal contact at the interface of the component parts of the machine. The mathematical model of the thermal process, which is built in the work, takes into account both the radial and axial effect of heat transfer in an electric machine. This model is presented in the form of a boundary value problem with impedance type conjugation conditions in a complex area. The modeling of the thermal processes of the windings and the air gap is very important for the analysis of the thermal field in an electrical machine. Therefore, it is important to identify these parts in an electric machine to study its temperature field. The entire area of the synchronous machines is divided into five types of simple subdomains, i.e. shaft, inner fan-shaped magnet, outer fan-shaped magnet, slot opening and slot. Moreover, on the border of the inner fan-shaped magnet and the outer fan-shaped magnet, the slot opening and the slot, we have an perfect thermal contact. Perfect thermal contact supposes that on the borders of the subdomains there holds an equality of the temperatures and heat fluxes. The boundary conditions of the fourth kind give essentially the rule of conjugation of the temperature fields of the subdomains of the body, which is investigated and the external body, in which heat is transferred by heat conduction. The problem of determining the temperature field in a multilayer cylindrical area with different thermal characteristics of the layers is solved by the finite element method.

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Axial Convection of Two Immiscible Fluids in a Vertical Tube

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The problem considered here is the two immiscible fluids motion with a cylindrical interface. The first time this problem was formulated by V. V. Puchnachev in 2012. Let the interface equation have the form $r = R_1 R_2$ where R_2 is the tube radius. The solution of the original Oberbeck-Boussinesq equations is sought in the form $u_j = (0, 0, w_j(r, t))$, $q_j = -A_j(r, t)z + T_j(r, t)$, $p_j = -B_j(r, t)z + q_j(r, t)$. Here $j = 1, 2$ is subscript that indicated fluid number. So the axial velocities in system arises under the combined action of the thermocapillary force and axial gradients of temperature and pressure. After substituting the form of the solution to the convection equations we obtain conjugate linear initial boundary value problem for functions $A_j(r, t)$, $w_j(r, t)$, $C_j(t)$. Here $C_j(t)$ is additional pressure gradients along the axis z, which are interconnected by the condition of normal stresses equality at the interface. For complete certainty of the problem necessary, the flow rate of the first fluid or the total flow rate of the two fluids takes into account. Thus, the arising problem for these functions is the inverse.

The following results were obtained: 1. The stationary solutions for two types of volume flow specification are found; 2. In Laplace images, the solutions to non-stationary problems are obtained in quadrature's, which contains Bessel functions; 3. It is shown that if the temperature on the solid tube wall evenly tends to a stationary value with increasing time, then the solutions of the non-stationary problem with increasing time tend to a stationary mode. The convergence velocities are determined; 4. On base of the Laplace transform the numerical inversion the calculations results of the velocity and temperatures evolution are given. It has been established that due to the change in the temperature gradient on the tube wall, it is possible to control convective movement in the layer and cylinder.

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Finite Element and Finite Difference Algorithms for Calculations of Electron Energy Spectra in the System of Isolated Donor-Quantum Dot

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The structures based on system of isolated donors and quantum dots are proposed as elements of various nanosized devices: quantum computers, quantum sensing devices, etc. In order to model the whole device, one needs to develop effective numerical methods for calculation states in elementary structure of donor-quantum dot. In this work, numerical modeling of electronic states in the structure isolated donor-electrically confined quantum dot is carried out. The system is described with the problem for stationary Schrödinger and Laplace (Poisson) equations. Energy spectra of donor electron and quantum dot are calculated using the finite difference and finite element methods. When developing numerical methods, the singularity at the donor location should be taken into account using cusp condition. Boundary conditions to calculate confining electric fields on unbounded domains are proposed. Critical characteristics of electron shuttling between the donor and the quantum dot are calculated using proposed methods. The results of the work can be used in designing new nanoelectronic devices.

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Drag Based Vertical Axis Wind Turbine Numerical Efficiency Evaluation

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Climate changes have led to a new approach of the energy situation of Europe, Asia and the Americas that face the effects of pollution. Thus, wind energy, considered clean energy - zero emissions of carbon and greenhouse gases, has become a viable option for increasing the life sustainability on Earth. The terrestrial surface irregular warmed by the sun, associated with the Earth's rotation motion leads to the appearance of large air mass movements, and because of that, the wind energy is also an indirect form of solar energy. Wind turbines convert the air currents kinetic energy into either electrical or mechanical energy. By using the CFD methods, it is possible to make quantifiable views of the flows associated with physical phenomena for which experimental tests would be very expensive if not impossible. In this paper a vertical axis wind turbine, Lenz type, axis was numerically analyzed. To determine the wind turbine efficiency, CFD methods were used with the Ansys Fluent software, analyzing two cases where the current velocity of 12m/s and 14m/s was varied. The unstructured meshes with quad elements were performed in Ansys meshing, taking into account the constraints imposed by using the k-omega SST turbulence model. Thus, the value of Yplus on the wall was 1 and the growth rate of two neighboring cells was 1.1. In terms of numerical results obtained from unsteady analyzes, using the URANS method, emphasis was placed on observing the vortexes evolutions from a blade and their influence on the others with impact on the wind turbine efficiency. Thus, the variations of the torque coefficients and variations of the vorticity magnitude for different blade positions are presented. For future work, the numerical results will be validated using an experimental model which will be tested in the aerodynamic wind tunnel.

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Foreign Exchange Rate Forecasting With Artificial Neural Networks

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Forecasting exchange rates is an important financial problem that is receiving increasing attention especially because of its difficulty and practical applications. Artificial neural networks (ANNs) have been widely used as a promising alternative approach for a forecasting task because of several distinguished features. Neural networks were originally developed in cognitive science and later were used in engineering for pattern recognition and classification. Neural networks are used because they can model nonlinear behavior in financial markets, in contrast to traditional linear models which are more restrictive. Neural networks can approximate any nonlinear function and are capable of dealing with "noisy" data. This study investigates the use of Nonlinear Autoregressive with Exogenous Input (NARX) neural network, in forecasting the exchange rate of the Euro against the US dollar.

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Multifractal Analysis of Plasma Irradiated Tungsten Alloy Samples

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If the world's energy demand grows at the same rate as in previous decades, sooner or later we face an energy crisis. One solution is fusion energy. The main problem with fusion energy is the choice of proper plasma facing material for the fusion chamber and the most effective evaluation of the durability of this material. The interior wall materials must withstand constant heat and particle flows that affect and damage the material. At present, tungsten is used as the internal wall material in plasma fusion plants. To characterize the behavior of plasma facing materials under intense plasma flows, the materials are currently analyzed using SEM (scanning electron microscope) images of the surfaces and cross-sections, estimating the role of different kinds of defects (e.g., droplets, cracks, blisters, etc). However, there is still no reliable method to compare the abundance and the role of different defects on the surface of the irradiated materials. In this research, we propose the multifractal formalism for characterizing the distribution and properties of the defects. In this study, we investigate the surface multifractality of two tungsten alloys (95% W, 1.66% Fe, 3.33% Ni and 97% W, 1% Fe, 2% Ni) and pure (double forged) tungsten specimens, using the box count method. The test specimens are irradiated with one and two series of deuterium plasma pulses, prior to the analysis in a irradiation mode similar to fusion devices. Then the SEM images are taken to perform further analysis. The box counting method allows defining material defects from SEM pictures and predicting material behavior during irradiation and multifractal spectra enable us to distinguish the samples irradiated with different heat and particle loads.

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On Solutions Forward and Inverse Problem Potential Geophysical Fields

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The potential geophysical fields — gravitational, magnetic, stationary thermal — satisfy to the equation of Laplace. Therefore the theory and methods of interpretation developed for one of this field have the universal character. This paper presents the new methods to solve direct and inverse problems of potential geophysical fields. We create the fast methods for solving forward problems and original methods for inversion. The inverse problems are typical ill-posed problems: its solution in the general case is not unique and unstably depends on the initial data. Hence it is necessary to seek solutions on sets of correctness, choosing reasonable models of an initial approximation. On the basis of the local corrections idea, we developed an iteration algorithm with adjusting regularization for stable solution of the inverse gravity problem and suggested an original method of layered density correction in a nonuniform parallelepiped. The method was realized in the computer system of data interpretation applied to real problems of seismic density modeling. All stages of interpretation and processing of the data were integrated into one programming product.

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On One Applicability Criteria for Methods of Approximate Control System Reachable Set Construction

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In control theory, one of the important problems is the problem of reachable sets construction since these sets are used as supporting structures for solving a number of control problems. Within the framework of the presented research we will consider a certain class of control systems whose dynamic is described by a system of differential equations. These control systems, in the general case, can be described by rather complex differential equations. Therefore, there is no universal method for exact reachable set construction. In this connection, many methods for approximate construction of reachable sets have been developed. As a rule, such methods are based on the transition to discrete representation of time and space. Among these methods we should mention the grid method based on the attraction of points to the nodes of a stationary regular grid and the grid method based on the exact coordinates and filtering. The accuracy of these methods depends on the values of the time step and grid cell size. The smaller the values, the more accurately the reachable set will be constructed. It is important to note that the problem of approximate construction of reachable set is a resource-intensive and in order to obtain an acceptable accuracy it is highly necessary to use a powerful supercomputer based parallel computing technologies. Implementation of these methods in the form of programs and further numerical experiments with the same parameters revealed for a number of systems a significant difference in reachable sets calculated by different methods. In general, we do not have an exact reachable set and therefore we cannot determine which of the methods give a reasonably accurate result. Moreover, some methods may produce sets that are too inaccurate for a given time step and cell size. Such sets cannot be used for further calculations. That gives rise to the problem of developing an applicability criteria for the methods of approximate reachable set construction (for given values of time step and grid cell size). This criteria should allow to assess indirectly the quality of the reachable sets calculated by such methods. Within the framework of the study the authors proposed such criteria. It is based on the procedure for constructing controls that lead the system to the points of approximately calculated reachable sets with subsequent evaluation of pointing errors. This criteria examines the execution of the main property of the reachable set, namely, the possibility of leading the system to the points of such set.

Acknowledgements. This work was supported by the Russian Foundation of Basic Research (grant # 18-01-00221).

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An Algorithm for Estimating Reachable Set of Control System under Uncertainties

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In this study, we consider the problem of external ellipsoidal estimation of reachable set and tube of the trajectories of the control system with uncertainties in the system matrix and in the initial states. It is assumed that the unknown initial states of the system belong to a given symmetric star-shaped set of a special type. The matrix of linear terms in the phase velocities of the system is unknown. The coefficients of the system matrix belong to the known compact in the corresponding space, that is, the dynamic of the system is complicated by the presence of bilinear components in the right-hand sides of the differential equations of the system. In this study, a new class of uncertainties in the parameters of the system matrix is considered. Under such constraints, the dynamical system is nonlinear, and the reachable set loses the property of convexity. We deal with star-shaped reachable sets and use for its description the Minkowski gauge functions. Such systems can be found in many applied fields, such as engineering problems in physics and economics, biological and ecological modeling for the cases when stochastic nature of errors is in doubt. The main results of this study develop a technique of set-valued estimation and consist in finding algorithm that allows one to construct external ellipsoidal estimates of the reachable sets for considered system. Algorithm for estimating reachable set of bilinear control system is illustrated by model examples.

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Applications of the Group Analysis Method to the One and Two-Dimensional Gas Dynamics Equations in Lagrangian Coordinates

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The presentation is devoted to a comprehensive analysis of the one- and two-dimensional gas dynamics equations of a polytropic gas in Lagrangian coordinates. The equations describing the flows of a polytropic gas are reduced to Euler-Lagrange equations. Symmetries of the Euler-Lagrange equations are analyzed. Noether's theorem is applied for constructing conservation laws.

The results were obtained in collaboration with V.A. Dorodnitsyn, R. Kozlov and E.I. Kaptsov.

Acknowledgements. This research was supported by Russian Science Foundation Grant No 18-11-00238 "Hydrodynamics-type equations: symmetries, conservation laws, invariant difference schemes".

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Automated Management System for the Temperature in a Pyrolysis Station Used for End-of-Life Tires Treatment

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Pyrolysis is a process that runs in an oxygen-free environment and at the same time a thermochemical decomposition occurs. This process can be used for sound treatment of End-of-Life tires (EOLT) – one of the most dangerous wastes in the world. Temperature of pyrolysis process is a determining parameter for both the flow rate of the process and the resulting products. This requires the construction of an automated heating process management system that facilitates operation, increases safety and reduces the need for a permanent operator presence on the site. An automated management system for a pyrolysis station with three cameras used for EOLT treatment is presented in this paper.

Keywords: Automatic control systems, Pyrolysis, End-of-Life tires

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Thermal and Electrical Characterization of Nanocomposites with Carbon Particles

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Polymers are widely used in industry and in our daily life because of their diverse functionality, light weight, low cost and excellent chemical stability. However, they have not yet reached their full potential, improving electrical and thermal conductivity of polymers would lead to their widespread use in electronics.

This article discusses the electrical and thermal properties of PLA matrix composites and GNP and MWCNT fillers.Composites with single and bifiller to the percolation threshold will be examined. Finally, we outline the main advances, challenges and outlooks for highly thermal and electrical conductive polymer nanocomposites.

Keywords: Thermal conductivity, Electrical conductivity, Nanocomposites, Polymers

Acknowledgements. This work is supported by H2020-MSCA-RISE-734164-3D project

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A Direct Simulation Monte Carlo Approach on Riemann Problems in Gas Dynamics

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The Riemann type of problems have been extensively studied and solved in the continuum regime using standard CFD methods. In this work, the Direct Simulation Monte Carlo (DSMC) method is employed to solve one- and two-dimensional Riemann problems at near-continuum flow regimes as well as at higher Knudsen number flows where the continuum assumption is no longer valid.

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Price Forecasting and Risk Portfolio Optimization

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Stocks are part of every company's capital. Trading with them could be accomplished through stock markets. There, if a businessman wants to increase his company's capital, he can offer its newly published shares to the investors, so they can trade with them. Nowadays, the stocks trading is very popular. That is why the problem of forecasting asset prices is of a great scientific interest. In this work are represented ARIMA (Autoregressive Integrated Moving Average) models for forecasting the stock prices. For every model the expected return of the shares is calculated and the variance of the rate of returns is analysed based on historic data. Quarterly data on stock prices of the four biggest banks in the United States, that are classified by total assets, are examined for the period 01 April 2014 - 01 April 2019. These banks are as follow: J. P. Morgan Chase & Co., Bank of America Corp., Citigroup Inc. and Wells Fargo & Company. An optimization problem is formulated, that is based on Harry Markowitz's model. The solution of this problem leads to finding the optimal risk portfolio for one period ahead and gives an estimate value of the expected return rate. Depending on the coefficient of risk aversion, a comparative analysis of the structure of a complete portfolio of risk and risk-free assets is made. Similar approach could be used for future developments on this subject as well as in the practice of financial managers of funds and investors. A Matlab programming code is developed, that gives the results for an optimal risk portfolio with n assets, where the input data is: expected rate of returns, their standard deviation, and the corresponding to them correlation matrix.

Keywords: Investment strategies, Risk portfolio optimization, Price forecasting, ARIMA, Matlab

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Machine Learning Based EEG Classification by Diagnosis: Approach to EEG Morphological Feature Extraction

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A hypothesis that spike morphological features contain information that can be used for epilepsy type detection by machine learning methods is discussed. Investigation of approach to EEG (electroencephalogram) spike morphological feature definition in relation to machine learning based EEG classification by diagnosis is presented in this study. Two approaches of defining EEG spike morphological features are investigated: A) numerically evaluating EEG spike geometric features, e.g., upslope, downslope; B) using 300ms of spike (without additional features extracted) for classification. Lists of spikes are used for the classification. Before start of the algorithm some basic preprocessing steps are taken: electric utility frequency (50Hz) is removed. The EEG classification by diagnosis algorithm consists of these main steps: 1) EEG spike detection by morphological filter; 2) EEG classification employing spike morphological features (employing discussed approaches) by diagnosis using machine learning based classification algorithms. Various classification algorithms (e.g., artificial neural network based classifier, AdaBoost, decision tree, random forest, extremely randomized tree, etc.) and their quality metrics are considered (e.g., accuracy, true positive rate, true negative rate, etc.) as well as results of k-fold cross-validation are investigated in this work. EEGs from children (3-17 years old) are classified in this work. The EEGs under classification are patients diagnosed with: I) benign childhood epilepsy; II) structural focal epilepsy. Current results show that best performance ($80\% \pm 2\%$) is exhibited by Extremely randomized tree based EEG classifier employing spike upslope and downslope data.

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Unsteady Reversed Stagnation-Point Flow of viscoelastic Maxwell fluid over a Flat Plate

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An unsteady incompressible viscoelastic two-dimensional stagnation point flow at a solid wall is studied. The simplest differential viscoelastic fluid model (i.e., the upper-convected Maxwell model) is used. A front or rear stagnation point on a plane boundary is considered, and a wide range of possible behavior is revealed when the solution at infinity is modulated in time by a specified factor. An interest in the problem was prompted by Petrova, Pukhnachev and Frolovskaya [1,2] recent study of unsteady incompressible viscous flow near stagnation point. This problem has been tackled in the literature by many (see for example [1,2,3]). The solutions of governing equations are found in assumptions that components of extra stress tensor are polynomials of spatial variable along solid wall. The velocity profiles are obtained by numerical integration of a nonlinear ordinary differential equation. The results of numerical simulations demonstrate that solution strongly depend on initial data. A counterflow zones in the initial data can disappear in finite time. When flow is modulated in time by a periodic factor, the resulting solution can be either the periodic or the blow-up.

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Evolution of Ultra-Short Dark Solitons in Single Mode Optical Fibers

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The development of modern optical communication systems is connected with the need to use more efficient information transfer technologies and to optimize existing ones to allow the evolution of laser pulses over long distances with minimal losses. A characteristic phenomenon associated with the nonlinear properties of optical fibers is the formation of solitary waves. The soliton regime of propagation of laser pulses is well-known effect in the field of optical communications and it has been studied for decades. There are different types of solitons regarding to the dispersion and nonlinearity of the waveguides. In our investigations we are interested of so called dark solitons. They correspond to the solutions of the nonparaxial nonlinear amplitude equation (modified Nonlinear Schrödinger equation) of light pulses, evolving in optical fibers with normal dispersion. Their intensity profile is characterized by a dip in a uniform background. Studies have shown that dark solitons are quite stable in the presence of noises and they spread more slowly in waveguides with losses compared to other types of solitons. In addition, dark solitons are less affected by the factors that have impact on the bright solitons. These properties give them an advantage in their potential application in modern communication systems. The main object of the present paper is the study of the nonlinear propagation of ultra-short dark solitons in dispersive optical fibers in the frames of the nonparaxial nonlinear amplitude equation. This type of equation governs the evolution of narrow-band, as well as broad-band laser pulses with few oscillations under the envelope. We are looking for a solution of that equation, describing the evolution of broad-band laser pulses in nonlinear single mode optical fibers with normal dispersion. It is found an exact analytical soliton solution.

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Geodesic Lines on the Cyclic Helical Surfaces

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At present various curved helical surfaces with generatrix in form of an arc of a circular is widely used in technique. A vivid examples of such helical surfaces can serve gears in Novikov gearing or helicoidal rotors in vertical wind turbines. However, cracks appear on the working surfaces in the process of operation, the development of which can lead to the destruction of both the parts and the whole structure. Detection and elimination of such cracks, which are known to pass along geodesics, is an important part of the design and manufacture of details. Goal of the work is determination of the geodesic lines on the cyclic helical surface by minimizing the curve. Several geodesics were built on such surfaces in a graphic editor to confirm the results.

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A Mathematical Model of the Human Thigh and Its Connection with the Torso

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The estimation of body segment parameters is important for the kinetic and dynamic analysis of human motion. For studying these an accurate modeling of the individual segments of the body is required. One specific problem in that respect that still lacks a satisfactory solution is to model the thigh-torso connection. The

point is that one normally models the torso and the thigh with relatively simple geometrical bodies that do not tailor continuously into each other. One shall also take into account the specific geometric properties describing the connections of both. Finally, the modeling shall be done in such a way that the mass of the torso as well as of the thigh is reproduced as closely as possible. The purpose of this work is to improve the mathematical modeling of the human thigh of the Bulgarian males [1], taking into account that the segment is dissected from the torso with a plane passing through the anterior superior iliac spine at an angle of 370 to the midsagittal plane [2]. In our previous research, the thigh was modeled as a frustum of a cone. In the current study, the thigh is modeled with the geometric body being a combination of a frustum of the circular cone on top of which is placed a part of a cylinder cut with a plane making an angle of 530 with respect to its base. This second part extends from anthropometric points omphalion - iliospinale. Using the mathematical model suggested, after deriving the corresponding analytical expressions needed, we calculate analytically and estimate numerically the mass-inertial characteristics of the human body segments: the volume, the mass, center of mass, and the principal moments of inertia. We present a comparison between the results obtained within this model with our previous results reported in [1], as well as with other data for Caucasian reported in the literature. The results obtained and the procedure suggested in the current work allow one to claim that more realistic modeling of the shape of the human thigh is proposed. The model can be actually used when one needs such mass-inertial parameters in problems appearing in rehabilitation, medicine (orthopedics and traumatology), sports, ergonomics, etc.

Acknowledgements. The financial support by the Bulgarian National Science Fund: Contract DN-07/5 "Study of anthropometric and mass-inertial characteristics of the Bulgarian men and women via mathematical models of the human body" is gratefully acknowledged.

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Fuzzy Recognition of Proteins in Population Genetics Electrophoresis Experiments

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The efficiency of using the method of fuzzy analysis developed by us in processing the data of gel electrophoresis of the tissues of earthworms is demonstrated. The experiment studied the effect of laser radiation on the change in the genotype of the worms' population. To confirm this effect, it was necessary to establish the presence or absence of new proteins within the framework of evidence-based biology, depending on the intensity and time of laser exposure. The use of our technique allows determining the presence and reliability of this effect.

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Application of Two-Dimensional Pade Approximants for Reducing the Gibbs Phenomenon

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The Gibbs Phenomenon significantly reduces the quality of images processing when they presented as a sum of harmonics, which is typical for most popular graphic standards. Distortion occurs on the border of a sharp change in the contrast of the image and lead to the appearance of false optical shadows. This reduces the quality of the analysis when processing the results of x-ray and sonar studies. We suggest using the two-dimensional Pade approximants method we developed earlier to reduce the Gibbs phenomenon for the harmonic two-dimensional Fourier series. The scheme of application of the method and its effectiveness are analyzed.

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Two- and Three-Dimensional Numerical Simulations of an Evaporating Liquid Layer on a Heated Substrate

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It cannot be expected that two-dimensional (2D) numerical results give an accurate representation of the three-dimensional (3D) physics of sessile drop or liquid layers evaporation when surface-tension-induced (Marangoni) flow is not symmetric. Also, a 2D model only considers the components of interface curvature lying in the computational plane, whereas the out-of-plane components are neglected. This means that surface-tension-driven hydrodynamic instabilities can usually not be captured in a realistic way. Developing a high resolution numerical method for the simulation of two-phase heat and mass transfer is a viable way of getting deeper insights into such phenomena, and as consequence to better understand them. A reliable and flexible numerical method is developed in the present paper in order to achieve this goal. The method is applied to the simulation of sessile drops and liquid layers evaporation phenomena.Within this framework, a numerical model is developed using the PHOENICS Computational Fluid Dynamics software, which permits a high flexibility and sustainability of the model. The Finite Volume discretization method is used to solve the governing equations of the problem. The method is developed in two- and three-dimension. A mass-conservative Volume-of-Fluid (VOF) interface tracking method is adopted to capture the position of the two-phase interface and its influence on the fluids flow. For the latter two accurate VOF methods have been developed the CICSAM and the THINC-WLIC methods.Marangoni, capillary forces, static contact angles and evaporation are also developed and included in the model. The developed model is applied to the evaporation of well-defined liquid layers where experimental data are available. We investigate the coupled physical mechanism during the evaporation of a circular pool containing FC-72 heated from below. The model accounts for mass transport in surrounding air, Marangoni and gravitational convection inside the liquid and heat conduction in the substrate as well as fixed and/or moving interface. We show that under some specific circumstances transition can occur from 2D axisymmetric to 3D patterns.

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Some Analytical Solutions for Magnetic Flux Distribution in Long Josephson Junction with Second Harmonic in the Current Phase Relation

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In the current work long Josephson junctions are being considered. Magnetic flux distribution is the physical measure for phase difference of the wave functions in the superconducting layers of the junction. The current phase relation, in most cases, can be regarded as an odd strict 2π -periodic function, and hence it can be presented in order of Fourier by sinus. It is well-known from the physical experiment that with a sufficient degree of precision, a number of physical systems are reliably described with the contribution of only the first two harmonics. The adequate mathematical model for the distribution of the magnetic flux is then the “double sine-Gordon” equation, with Neumann’s boundary conditions at the ends of the junction. Even in the stationary case, the boundary problem is highly nonlinear and the only tool for its comprehensive study is numerical methods. The aim of the present work is to show that in the case of zero external current, the stationary equation turns out to be a fully integrable model, derived from a variation principle with a cosine. In our work analytical solutions for the magnetic flux distributions described in the terms of Jacobi’s elliptical sinuses are derived. Analytical studies in this case serve to further numerically study of this multiparametric nonlinear boundary task problem, which is so important in the applied nanophysics.

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Identification of the Elastic Modules of a Fibrous Composite by Solving Inverse Problems

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Generalized rheological method is used to construct constitutive equations of fiber composite materials with the fibers having high tensile stiffness and low stiffness upon compression. Polymer-based composite reinforced by thin carbon fibers used in the aerospace industry was chosen as the material for research. The method for determining the effective moduli of elasticity based on the analysis of the bending state of a thin rod was implemented to check the material's moduli. Photos of the bent rod with cantilever bending were processed to obtain a flat projection then digitized using computer programs. The deflection data along the rod length was used further in solving the inverse problem of determining flexural stiffness (it was considered constant) using the least squares method. Using Matlab functions, the problem of minimizing the standard deviation of the calculated deflection from the digitized one was solved on a discrete system of points along the axis of the rod. The deflection was computed by the difference method based on the specify Euler elastic equation which takes into account the flexural stiffness, deformation along the rob, Timoshenko effect (the effect of shear deformation) and Cosserat effect (the effect of independent turns of the reinforcing fibers relative to the matrix). The Young modulus upon tension was measured experimentally using standard techniques. The Young's modulus upon compression was calculated using the value of flexural stiffness corresponding to the best approximation of calculated deflection to the digitized deflection. As a result, it was shown that the ratio of moduli for the material under study is in the range of 50-60% and if the calculation of flexural stiffness is made with Young's modulus under tension, it is possible to obtain a relative error in the deflection up to 30% with the increased thickness of the rod.

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Geometry of Riemannian Spaces of the 2nd Approximation

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For the Riemannian space, the invariantly associated space is constructed, which implements a second order approximation for the primary Riemannian space.

We studied Lie groups of infinitesimal motions and conformal transformations in the associated space, when the initial space is a space of nonzero constant curvature.

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Numerical Simulation of Nonlinear Development of Perturbations in a Thin Layer of a Viscous Liquid

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Thin layers of viscous liquid (liquid films) moving on a solid surface under the action of gravity for moderate Reynolds numbers at heat and mass transfer (evaporation, condensation) are investigated. The relevance and practical importance of studying the flows of thin layers of viscous liquid is associated with their wide implementation in numerous heat and mass transfer apparatus of heat power, chemical, metallurgical, food, pharmaceutical industry (film rectifiers, columns with a flat-parallel nozzle, absorbers), etc. The combination of a small film thickness and a large contact surface can significantly intensify chemical, thermal, diffusion processes. A nonlinear partial differential equation for the deviation of the free surface of the liquid film from the undisturbed state in the processes of heat and mass transfer is presented where x is the spatial coordinate, t – the time. The coefficients of the equation include the parameters: surface tension, surface viscosity, Marangoni parameter, constant shear stress. Within the framework of this model, computational experiments were carried out to calculate the wave characteristics of the liquid film: frequency, increment, phase velocity in the processes of evaporation and condensation, as well as computer simulation of the free surface of the vertical water film.

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Numerical Modeling of the Mass of the Flowing Liquid at Transverse Oscillations of the Straight Tube

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In this research, a problem of determining spatial vibrations of straight tube is considered. The tube is rigidly fixed at both ends. The vibrations appear under the impact of fluid flow on the tube sides and the additional shock pulse. The mathematical model includes the fourth-order PDE with coefficients depended on the liquid parameters. The numerical method for determining characteristics of spatial vibration is proposed. The method is based on using of finite-difference equations and a regularization technique. The computational scheme involves the calculating amplitudes and the phase difference of vibration at the tube points by the Fourier transforms. In order to evaluate the reliability of method the computational experiment was carry out. The computational results show the suffusion accuracy of method.

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Singularities in Viscous Flows

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Origination of singularities in viscous incompressible fluid flows is studied on the basis of exact and asymptotic solutions of the Navier-Stokes equations. Examples of stationary solutions of these equations with point or distributed singularities are presented. The asymptotic character of singularities, such a source, drain, point vortex, or a combination of these singularities is elucidated. Examples of unsteady motions, which demonstrate the change in the flow domain topology with time or change in the structure of streamlines, are considered. Problem of deformation of a strip or layer bounded by two free boundaries or a solid wall and a parallel free surface are studied in detail. Conditions of blow up of the solutions of the corresponding initial-boundary value problems within a finite time are found.

Acknowledgements. This work is supported by Russian Foundation for the Basic Research (grant No. 19-01-00096).

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Stokes Problem for Second-Order Liquid

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Let the liquid fill the space outside the cylinder, which makes longitudinal harmonic oscillations. The resulting problem for second-order fluid has a periodic time decision. The field of velocities and pressures is found in a wide range of governing parameters – the Reynolds number and relaxation parameter [1,2]. Unlike the second Stokes problem for an ordinary viscous fluid [3], where the pressure is constant, in a second-order fluid it oscillates at twice the frequency. This effect can be used to identify a model which is used in describing the motion of aqueous solutions of polymers [4,5].

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Spline Approximation with High Accuracy

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In this article, we discuss the issue of the best uniform approximation with high accuracy. The best uniform approximation of degree 5 is considered to special curves and is given in explicit form. The approximation is constructed so that the error function is the monic Chebyshev polynomial and the error function has the highest number of equioscillation times.

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Risk Estimation in Case of Limited the Insurance Liability

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There are a number of models, used for estimating the insurance risk. In practice are divided two directions of analysis called "Classical risk theory" and "Modern risk theory." The modern risk theory includes additional conditions, typical for the insurance company business like taxes, different internal costs and many others. Unfortunately these peculiarities are usually not available for the "outside world." That's why, it is very difficult to do analysis with such details. On the other hand, the classical risk theory is focused on the analytical models of stochastic processes which open a wide field for mathematical application.

As a fundamental part of insurance risk theory, the model of Cramer-Lundberg is based on the balance between claims costs of the insurer and the premium payments from the side of the insured person. The model also include information about the retention and the initial capital necessary to meet the expected claims costs. The expected claims process is a compound stochastic process, which is usually modeled by a continuous distributions. Often used approach for reducing the insurance risk is by using franchise value or just declaring a limit value for the insurer's liability. Including such restriction in the models the claims cost distribution is continuous no more. This involves considering of appropriate approximations for the mixed discrete-continuous distributions of the claims cost. Also all estimations about the level of risk like the retention and the necessary free reserves are affected by the choice of approximate distribution.

In the current work is considered the influence of different transformations over the random variables, which describe the claims cost in the risk model. For achieving the estimations of the retention and the free reserves we use the classical risk model of Cramer-Lundberg for a fixed period of time for one year. The experiment we provide is based on empirical distribution, which we use for simulating transformed random variables using Fourier approximation for the mixed discrete-continuous probability distributions. Finally, there is a comparison between the results for the estimated free reserves without limited insurance liability and after including the liability limitation. The considered approach uses easy methods for implementation and could find useful application in insurance practice.

Acknowledgements. This paper contains results of the work on project No 2019-FNSE-05, financed by Scientific Research Fund of Ruse University.

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Computation of Risk in Pricing of Investment Projects

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The notion of risk in the economy and finance refers to the deviation in one or more returns of one or more future events from their expected value. The value of these results may be positive or negative. Positive risk is seen as an opportunity, and the general use of the word risk only focuses on the potential damage (loss of positive outcomes) that may arise from a future event that results either from incurring costs (downside risk) or disability to make a profit (upside risk). The negative consequences of risk determine the need for risk study and management. The report presents the most commonly used one-step decision-making criteria in the context of risk uncertainty: Laplace's Criterion – a widely used criterion in decision-making tasks in uncertainty based on the principle of insufficient justification; Gurvish's Criterion – a Laplace Criterion analogue, but implements the estimation of the mathematical expectation of loss in a different way; minmax / maxmim criterion – the realization of this criterion assumes choosing the best and worst opportunity; Seaview's criterion – a less pessimistic criterion from the previous one.

The "scenario analysis" approach is used, in which the financial analyst requires the technical project manager to select indicators in case of poor circumstances (low sales volume, low prices, high cost, etc.) and good circumstances. Based on the latter, net NPV (Net Present Value) is calculated and compared, i.e., the future value of net income and the present value. A fundamental principle of assessing the

cost-effectiveness of an investment project corresponds to the economic behavior of a person taking decision about the appropriateness or loss of certain investments. They are expressed by the indicator of net income. When calculating the net income, the results and losses are always compared. Examples of different scenarios and selection criteria for risk estimation for particular projects are considered. The advantages and disadvantages of the different options are discussed.

Keywords: Risk, downside risk, upside risk, risk assessment, risk premium, discount, scenario analysis

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Users Traffic on the Two-Sided Internet Platforms. Qualitative Study

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The volume of users on Internet platforms, their characteristics, dynamics and equilibrium have mainly been studied with game-theoretic approach and with statistics. However, growing number of Internet platforms with large amount of data allows to construct differential equations models with the help of statistical techniques, and then to apply the dynamical systems theory for the qualitative study of the phase portraits and for understanding the underlying laws that govern the dynamics. The reconstructed model of the two-sided Internet platform's traffic estimates the volume of users from each side of the platform. The model is based on the negative same-side and positive cross-side network effects. The cross-side network effect is represented by a real-valued attachment function expressing interest of one type of users in the opposite type of users. I will discuss theorems describing the long-term behavior, tendency and equilibrium of the users' volume on the platform. These results were obtained with the help of classical theory of dynamical systems. I will present simulations of some examples and show how attachment functions influence users' dynamics. The dynamical system's approach to the study of two-sided platform allows natural generalization to multi-sided platforms (MSP), where one can utilize well developed theory of multidimensional dynamical systems and obtain results for MSP with high number of dimensions. The dynamical systems approach allows to see the effects of externalities on the trajectories of the volume of users

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The Wake Vortex Dynamics above the Underlying Surface of Different Types

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The study of the evolution of the vortex wake behind an aircraft near the surface is an important problem for flight safety. Of particular interest is the behavior of the vortex wake, when the aircraft is flying at altitudes less than wing span. In this case, the vortex wake interaction with the boundary layer is essential. There are a considerable number of airports where the aircraft flies over the water surface when landing. The behavior of the vortex wake under these conditions has been poorly understood. In this paper, using numerical modeling, a comparative analysis of the vortex wake dynamics above the soil and water surface is carried out. The modeling method is based on the direct numerical solution of the equations for the turbulent motion of a viscous fluid. A numerical method with a second-order accuracy difference scheme is used. The results of the calculation of the wake vortex of the wide-body aircraft A-380 are presented. The cases of the flight of the aircraft in the flight and landing configurations at different heights above the solid and free surface are considered. The differences in the mechanisms of the viscous interaction of a vortex wake with a boundary layer on surfaces of different types are shown.

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Ellipsoidal Vortices in Compressible Rotating Fluid

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We study vortices in 2D polytropic uniformly rotating compressible fluid within the class of motions with uniform deformation. In the Eulerian coordinates, it implies that the velocity is a linear function of coordinates and the level lines of the pressure are ellipses. It is shown that this class of solutions is completely defined by a system of quadratically nonlinear ODEs of a higher order. Under certain assumption this system is integrable. In particular, it happens for the adiabatic index equal to 2. Formally this case corresponds to the case of shallow water on a rotating plane. The equilibria of this system form two families. One of them is one-parametric, it corresponds to a vortex, and the parameter is the intensity of vortex. We show that the nonlinear stability of these steady vortices for the physically meaningful adiabatic indices depends only on the ratio between the parameter of intensity of the vortex and the Coriolis parameter. It is shown that if the rotation of the coordinate frame presents, the domain of stability exists both for "anticyclonic" and "cyclonic" cases, nevertheless it shrinks if the Coriolis parameter tends to zero. Another family is two-parametric, it corresponds to a shear, and the equilibria are always unstable. Both families of equilibria are prototypic for elementary atmospheric structures like cyclone/anticyclon and trough/ridge.

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Modeling Perspectives of Forest Growth and Yield: Framework of Multivariate Diffusion Process

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Today's approaches to modeling of forest trees and stands are in most cases based on that the models are divided into static regression models describing individual trees and stands variables. This study proposes a general stochastic dynamical model of a forest stand with the object to include random forces governing the dynamic of multivariate distribution of tree (stand) size variables. The dynamic of the multivariate probability density function of tree size components (diameter, height, crown base height, crown width and so on) in a stand is described by a mixed

effect parameters Vasicek-type multivariate stochastic differential equation (SDE). The advantages of SDE method are that it do not need to choose many different equations to be tried, it relates the tree size components dynamic against the age dimension (time), and consider the underlying covariance structure driving changes in the tree (stand) size variables. SDE model allows us a better understanding of biological processes driving the dynamics of natural phenomena. The new derived multivariate probability density function and its marginal univariate, bivariate and trivariate distributions, and conditional univariate, bivariate and trivariate distributions can be applied for the modeling of stand attributes such as the mean diameter, height, crown base height, crown width, volume, basal area, slenderness ratio, their increments and much more. This study introduces general multivariate mutual information measures based on the differential entropy to capture multivariate interactions between state variables. The purpose of the present study is therefore to experimentally confirm the effectiveness of using multivariate mutual information measures to reconstruct multivariate interactions in state variables. In this regard, the SDE model was fitted using measurements obtained from permanent experimental pine-dominated stands.

Keywords: Multivariate Vasicek-type stochastic differential equation; marginal distributions; conditional distributions; entropy

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Supercomputing Analysis of Seismic Efficiency of the Electromagnetic Pulse Source "Yenisei"

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Computational technology for multiprocessor computing systems of cluster architecture is developed for numerical modeling of wave fields generated by the electromagnetic pulse source "Yenisei" in blocky-layered geomedia. To describe the wave processes, mathematical models of the dynamics of elastic, viscoelastic and elastic-plastic media, of porous and granular materials are applied. The algorithms of numerical realization are constructed based on the method of two-cyclic splitting with respect to spatial variables. Computational experiments showed that the proposed technology allows reproducing the system of waves near the region of excitation of seismic oscillations in 3D setting with a high degree of details and accuracy. The main goal of our research is to optimize the geometric and mechanical characteristics of the source based on mathematical modeling of the propagation of waves generated by the source in layered soil massifs of complex rheology. We applied this technology to the analysis of frequencies and amplitudes of waves generated in the near-surface soils, and showed that the source "Yenisei" possesses the required seismic parameters. Given contribution presents the results of computation of seismic efficiency of the electromagnetic pulse source as the ratio of the energy passing through the reflecting surface in the depth of layered massif to the energy of pulse effect on the surface. The applied method is based on the analysis of the Umov-Poynting vector field obtained by means of high-performance computations. The results show that, by the criterion of useful seismic energy, pulse sources are not inferior to sources of vibratory type.

Acknowledgements. The reported study was funded by RFBR, Government of Krasnoyarsk Territory, Krasnoyarsk Regional Fund of Science to the research project No. 18-41-242001.

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Analysis of Structural Changes of Seismic Wave Fields in Cracked and Fluid-Saturated Media

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The paper is based on the multidisciplinary approach within the problem of active seismic monitoring the processes of crack formation and dilatancy developing in seismological zones. An additional approach to the dynamic characteristics of the wave field is proposed for the previously proposed approach to tracking such processes by changing the anisotropy coefficients of the medium and the propagation rates of longitudinal and transverse waves. They are associated with the allowance for variations in waveforms and the nonlinear transformation of wave fields due developing geodynamic processes in focal zones. This approach is substantiated by the results of experimental studies on the vibration sensing of mud volcanoes of the Taman mud-volcanic province and a tectonic fault in the Novosibirsk region. The effectiveness of these wave field parameters to solve the problem of active seismic monitoring is shown. At the same time, the problem of estimation the dynamics of the development of geodynamic processes in the focal zone is the multiparameter problem.

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Higher Regularity of Solutions of Singular Parabolic Equations with Nonstandard Growth

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We present new results on the global regularity properties of solutions to parabolic equations which involve $p(x)$ and $p(x,t)$ Laplace operators. The solutions of such equations are usually understood in a weak sense. In particular, the time derivative is a distribution which does not belong to any Lebesgue space. We find conditions on the data that guarantee the existence of strong solutions. For these solutions, the second derivatives in space and the first derivative in time belong to Lebesgue spaces with variable exponents prompted by the equation. Moreover, it is shown that under certain conditions on the data the solutions of the parabolic problem are continuous with respect to time in the sense of Holder and Lipschitz.

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Numerical Modeling of the Dynamic Processes in Liquid Crystals under the Action of Thermomechanical and Electrostatic Perturbations

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Liquid crystal is an intermediate state of matter, which appears at the same time the properties of elasticity and fluidity. The mobility of molecules allows their orientation to be changed by external forces and, thus, to control their properties. Mathematical model for description of the dynamic processes in liquid crystals under weak mechanical, temperature and electric perturbations was proposed in our works earlier. Given contribution presents the algorithm and parallel program system for numerical implementation of this model in 2D case. Basic system of equations of the model is solved using the method of two-cyclic splitting by spatial variables. At the stages of this method, 1D subsystems of the acoustics equations for liquid crystals and related equations of thermal conductivity in spatial directions are solved with the help of Godunov's gap decay method, Ivanov's method with controlled energy dissipation and Crank–Nicolson's scheme. The right-hand sides of equations in the acoustic approximation of a liquid crystal arise under the action of electric field, for finding of which we use the method of straight lines. Numerical algorithm is implemented using the CUDA technology for computer systems with graphics accelerators. Main stages of the algorithm are executed sequentially, the parallelization of computations is performed inside each of the stages. Parallel program contains modules realizing the Godunov, Ivanov and Crank–Nicolson schemes at the stages of splitting method, and also the method of straight lines with the use of a three-point sweep, the Fourier transform and the SLAE solution method by means of the LU-decomposition, the iterative method of solving the equation for electric potential. Some computations, demonstrating the change in orientation of the molecules of a liquid crystal under the influence of external forces, are performed.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (project no. 18-31-00100).

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Bernoulli-Trials Approximations of the Collision Process in the DSMC Method

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The binary collision process in rarefied gas is presented by a general transition operator transforming the gas state form one state to other. Its approximations with respect to time step are considered and critically analyzed with respect to their application to gaseous microfluidic problems. Some numerical examples will be given showing that the general Bernoulli-trials collision scheme covers the properties of all specific approaches used by now.

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Two Zaslavsky Maps in Pseudorandom Byte Generation

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We present a novel, two Zaslavsky maps based pseudo-random byte formation function. We evaluated output data theoretically and tested by NIST, DIEHARD and ENT statistical suites. The results of the cryptographic analysis show that the output bytes behaves like an ideal random source.

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Self-Regulation of the Labor Market: Dynamic Optimization Model

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In this article, the authors put forward a hypothesis that the labor market is a self-regulating system which under certain assumptions can be expressed through such indicators as the unemployment rate and the GDP. The article also includes an overview of research literature on self-regulation of the labor market, in particular the econometric model based on the idea of wages being used to balance supply and demand. In the second part of the article the authors present the dynamic mathematical model for optimization of specialist training at the university. The dynamic model consists of a phase vector, which is formed with the help of several key parameters describing the process of specialist training at the university (education programs, enrolment capacity, facilities, indicators of the social sphere and so on), and a control vector, which describes a set of technologies used for organization and realization of specialist training at the university (various methods of specialist training, funding, and so on).

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Levy-Flight Salp Swarm Algorithm for Function Optimization and Engineering Design Problems

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The Salp Swarm Algorithm (SSA) algorithm is a bio-inspired optimizer. Aiming at the phenomenon that Salp Swarm Algorithm has slow convergence and low precision, an improved version of SSA algorithm based on Levy-flight strategy, which is named as LSSA, is proposed. Levy-flight can increase the diversity of the population against premature convergence and make the algorithm jump out of local optimum more effectively.This approach is helpful to obtain a better trade-off between exploration and exploitation ability of SSA, thus, which can make LSSA faster and more robust than SSA. The proposed LSSA algorithm is applied on several benchmark functions and engineering design problems. The results show competitive results of LSSA concerning the other metaheuristic algorithms.

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Existence, Continuation, and Lower Mass Bounds for the Landau Equation

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Kinetic equations model gas and particle dynamics, specifically focusing on the interactions between the micro-, meso-, and macroscopic scales. Mathematically, they demonstrate a rich variety of nonlinear phenomena, such as hypoellipticity through velocity-averaging and Landau damping. The question of well-posedness remains an active area of research. In this talk, we look at the Landau equation, a mathematical model for plasma physics arising from the Boltzmann equation as so-called grazing collisions dominate. Previous results are in the perturbative regime, or in the homogeneous setting, or rely on strong a priori control of the solution (the most crucial assumption being a lower bound on the density, as this prevents the elliptic terms from becoming degenerate). We prove that the Landau equation has local-in-time solutions with no additional a priori assumptions; the

initial data is even allowed to contain regions of vacuum. We then prove a "mass spreading" result via a probabilistic approach. This is the first proof that a density lower bound is generated dynamically from collisions. From the lower bound, it follows that the local solution is smooth, and we establish the mildest (to date) continuation criteria for the solution to exist for all time.

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Numerical Approach for a Class of Differential Equations with Constraints

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Frequently, numerical solutions to evolution differential equations with constraints are computed on artificial space cutoffs because of the necessary boundedness of computational domains. Therefore, well-posed boundary conditions are needed at the artificial boundaries. Moreover, these boundary conditions have to be chosen in such a way that the numerical solution of the cutoff equation approximates as best as possible the solution of the original problem on the infinite domain, and this includes the preservation of constraints. In this talk, we consider well-posed, constraint-preserving boundary conditions for a vector-valued wave equation with constraints. Then, we construct an equivalent extended problem, which includes the constraints as dynamical variables. Because of the constraints directly entering the evolution, the extended problem may present a preferable alternative for numerical approximation.

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A Discrete Mathematical Model for Single and Collective Movement in Amoeboid Cells

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We propose a new discrete mathematical model for individual and collective cell motility. We introduce a mechanical model for the movement of a cell on a two-dimensional rigid surface to describe and investigate the cell-cell and cell-substrate

interactions. The cell cytoskeleton is modeled as a series of springs and dash-pots connected in parallel. The cell-substrate attachments and the cell protrusions are also included. In particular, this model is used to describe the directed movement of endothelial cells on a Matrigel plate. We compare the results from our model with experimental data. We show that cell density and substrate rigidity play an important role in network formation.

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Solving an Elliptic Problem with Regular and Parabolic Boundary Layers by a Multigrid Algorithm

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A two-dimensional linear elliptic equation with parabolic and regular boundary layers is considered in the unit square. It is solved by using an upwind difference scheme on the Shishkin mesh which converges uniformly with respect to a small perturbation parameter. The scheme is resolved based on an iterative method. It is known that the application of multigrid methods leads to essential reduction of arithmetical operations amount [1] and the references therein. Earlier in [2] we investigated the cascadic two-grid method with the application of Richardson extrapolation to increase accuracy of the difference scheme uniform with respect to a perturbation parameter, using an interpolation formula uniform with respect to a perturbation parameter. We obtained that the usage of the auxiliary mesh with the number of nodes in two times less than the initial mesh leads to increase accuracy of the difference scheme by an order uniform with respect to a perturbation parameter. In this paper multigrid algorithm of the same structure is studied. We also used an interpolation formula uniform with respect to a perturbation parameter. The application of the Richardson extrapolation method based on numerical solutions on the last three meshes leads to increase accuracy of the difference scheme by two orders uniform with respect to a perturbation parameter. We compare the proposed cascadic multigrid method with a multigrid method with V-cycle with a special restriction operator as [3]. The results of some numerical experiments are discussed.

Acknowledgements. Research has been supported by the program of fundamental scientific researches of the SB RAS No I.1.3., project No 0314-2019-0009.

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A Comparison of Monte Carlo Methods Based on Faure and Sobol Sequences for Multidimensional Integrals in Air Pollution Modelling

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Air pollution and meteorological models are in front places among the examples of mathematical models with a lot of natural uncertainties in their input data sets and parameters. Sensitivity analysis is a powerful tool for studying and improving the reliability of such models. In this work some results of the global sensitivity study of the Unified Danish Eulerian Model (UNI-DEM) have been presented. One of the most attractive features of UNI-DEM is its advanced chemical scheme — the Condensed CBM IV, which consider a large number of chemical species and numerous reactions between them, of which the ozone is one of the most important pollutants for its central role in many practical applications of the results. A comprehensive experimental study of quasi-Monte Carlo algorithms based on Faure and Sobol sequences for multidimensional numerical integration has been done. The algorithms have been successfully applied to compute global Sobol sensitivity measures corresponding to the influence of several input parameters on the concentrations of important air pollutants. The study has been done for the areas of several European cities with different geographical locations. This is the first time when Faure low discrepancy sequence is applied to this problem and a comparison with the low discrepancy of Sobol has been made. The numerical tests show that the stochastic algorithms under consideration are efficient for the multidimensional integrals under consideration and especially for computing small by value sensitivity indices.

Acknowledgements. This work is supported by the Bulgarian National Science Fund under Bilateral Project Bulgaria-Russia DNTS 02/12-2018 "Development and investigation of finite-difference schemes of higher order of accuracy for solving applied problems of fluid and gas mechanics, and ecology" and by the Bulgarian National Science Fund under Project DN 12/5-2017, "Efficient Stochastic Methods and Algorithms for Large-Scale Problems."

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Assessment of Optimal Tool Parameters used in Numerical Simulation for Hot Mandrel Bending of Pipe Elbows

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This paper is tackling the elbows bending process using hot mandrels. Through this process the pipe is pushed over a fix or mobile mandrel, which defines the expansion and the radius of curvature and is very important that mandrel deformation to be minimum. The paper presents the finite element model developed for the process described above, the optimization of tool parameters used to obtain and validate the simulation conditions in order to evaluate the effect of different materials on mandrels performances. The numerical simulation analyses were carried out by finite element analysis software ANSYS LS-DYNA which has a vast array of capabilities to simulate extreme deformation problems. Results obtained following the manufacturing process FEM simulation optimization are expected to reduce the mandrel production and exploitation costs.

Acknowledgements. This work was carried out within POC-A1-A1.2.3-G-2015, ID/SMIS code: P_40_422/105884, "TRANSCUMAT" Project, Grant no. 114/09.09.2016 (Subsidiary Contract no. 3/D.1.1/114/27.11.2017), Project supported by the Romanian Minister of Research and Innovation.

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Using Smart Contracts for Software Lifecycle Management Automation

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The rapid development of the DLTs in the recent years solved many of the shortcomings of original Bitcoin network and extended the areas and problems they were used to address. As result one of the new issues that architects have to decide on is which DLT is best suited for the specific tasks they are solving. In this article different DLTs and DLT features will be reviewed regarding their suitability to solve current problems in the area of the software lifecycle management (SLM). Software lifecycle management is the process of managing procedures that are executed at customer site, in their cloud, IoT devices and for edge computing landscapes. Some of these procedures are software products installation, upgrade, transport customized code and data, system copy, configuration, and many other. In the Enterprise world these procedures may be very complex as they involve coordination and configuration for usage of many tools and many different parties on different landscapes and clouds. Often many involved parties need to cooperate to successfully complete complex set of mutually dependent software lifecycle management procedures with minimal risk, downtimes and resource usage. Some of the parties that could be involved in a complex software lifecycle management project are customers, software providers, consulting partners, hardware vendors, and software vendors. These parties are typically not hierarchically organized and may not fully trust each other as they may have contradicting interests for specific aspects of the process. Their goals are often project-based, temporary, and not strictly formalized in detailed legal frames. The success of the project often depends on the results of their collaboration and proper sharing of efforts, knowledge and resources. This makes usage of a DLT a good candidate to handle and automate the collaboration and sharing aspects of their interactions by introducing additional trust between parties. In this session a DLT-based system will be described that could streamline and automate processes that are currently manual and time consuming. Having a DLT in place provides each party with own copy of the information. This makes it independent of the future relationship between involved parties.

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Simulation of Thermal Effects of Engineering Objects and Climate Changes on the Permafrost Boundaries

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Permafrost occupies about 25% of the total land area of the globe and is highly susceptible to external influences caused by human activity and climate change. Most engineering structures and buildings in the permafrost zone use the principle of preserving the frozen state of the soil. For these purposes, various options for thermal insulation of the soil surface, piles, which are the foundation for residential buildings, cooling devices for thermal stabilization of the soil and other options for preventing permafrost degradation, are used. Permafrost thawing can lead to serious accidents and the destruction of buildings. For example, damage to production wells in the northern oil and gas fields due to permafrost processes leads not only to large financial losses, but also to be environmental consequences for the environment. The paper discusses various mathematical models for describing the distribution of thermal fields in the surface layer of soil from various sources of heat, or cold, which, together with seasonal climatic changes, form unsteady thermal fields in the soil. The developed algorithms are focused on high-performance computers and were used in the design of various oil and gas fields located in the permafrost zone. The main attention is paid to the adequate setting of boundary conditions and maximum consideration of various parameters, including technical characteristics of engineering objects, climatic conditions and soil lithology in the field of modeling thermal fields. The presented numerical calculations show the possibility of using these models and algorithms for long-term forecasting of the development of thermal processes in the soil. Minimization of heat exposure in the system heat sources (cold) – permafrost will avoid accidents at oil and gas fields associated with changes in temperature conditions in the soil, and increase the stability of the building with pile foundations.

Acknowledgements. The work was supported by Russian Foundation for Basic Research 19-07-00435.

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Numerical Modeling of Seismic Wave Propagation Generated by Electromagnetic Pulse Source in Fractured Medium

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We consider the problem of increasing the efficiency of seismic exploration using electromagnetic pulse source by adjusting its mechanical and geometrical characteristics based on mathematical modeling of seismic wave propagation generated by the pulse source in medium with complex rheological properties. To describe wave processes in structurally inhomogeneous fractured medium, the model of block medium consisting of elastic blocks and fractured interlayers is proposed. It is assumed that the cracks in medium propagate through the interlayers only. When the critical level of deformation is reached, a crack occurring in the interlayer alters its properties. To take into account dynamic processes in interlayers, rigid contact, i.e., a rheological element usually applied for the modeling of medium having different resistance to tension and compression, is used. Thermodynamic consistency of the proposed model is shown. For numerical implementation of the model, a parallel computational algorithm is developed. The algorithm is based on the two-cycling splitting method in combination with the monotone ENO-scheme describing wave propagation in blocks and the non-dissipative Ivanov's scheme for the interlayers. Parallelization of computations is performed at stages of the splitting method. A series of numerical computations is performed to show the wave fields generated by the electromagnetic pulse source in fractured medium.

Acknowledgements. This work was supported by the Russian Foundation for Basic Research (project no. 18-31-00100) and by the Russian Foundation for Basic Research, Government of Krasnoyarsk Territory, Krasnoyarsk Regional Fund of Science (project no. 18-41-242001).

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Seasonality of the Levels of Particulate Matter PM10 air pollutant in the city of Ruse, Bulgaria

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This paper presents empirical study of the seasonality in air pollution of a Bulgarian city, caused by PM10 (particulate matter 10 micrometers or less in diameter). Different statistics, describing the change of PM10 over time have been considered. The trend and seasonality in the data are modeled using different approaches of time series analysis. The results are used to obtain point and interval estimates for future values of PM10 levels.

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Numerical Simulation of Alternative Fuels Combustion in Gas Turbines

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The paper presents the thermodynamic analysis for changing the fuel of an aviation gas turbine from kerosene to alternative gaseous fuels. Theoretical results are obtained using the CEA program and the numerical ones using a commercial CFD code. The gas turbine used for the application is a helicopter turboshaft for which the geometry of the combustor was carefully reproduced in a CAD environment. The paper details each necessary step to obtain the numerical results, as well as some comparison with experimental data. The conclusions are related to the similarities and the differences between the experimental data, considering the complexity of the entire gas turbine.

Acknowledgements. This work was carried out within POC-A1-A1.2.3-G-2015, ID/SMIS code: P_40_422/105884, "TRANSCUMAT" Project, Grant no. 114/09.09.2016 (Subsidiary Contract no. 1/D.1.5/114/24.10.2017), Project supported by the Romanian Minister of Research and Innovation.

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A Momentum Preserving Numerical Method for a Sixth Order Boussinesq Equation

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In this talk a momentum preserving scheme is constructed and studied for the Sixth Order Boussinesq Equation (SOBE)

$$\frac{\partial^2 u}{\partial t^2} = \Delta u + \beta_1 \Delta \frac{\partial^2 u}{\partial t^2} - \beta_2 \Delta^2 u + \beta_3 \Delta^3 u - \Delta f(u),\ x \in (-\infty, \infty)\,,\ t > 0.$$

The momentum preserving scheme is based on the representation of the SOBE as a Hamiltonian system. The finite difference scheme is explicit, conditionally stable and second order accurate in space and time. We prove that the discrete solution conserves *exactly* the discrete momentum and the discrete mass, and *approximately*, with $O(h^2 + \tau^2)$ error, the discrete Hamiltonian (energy). Numerical experiments are provided for quadratic and cubic nonlinearities. The numerical results show good agreement with our theoretical results.

Acknowledgements. This work is partially supported by the Program for young scientists and postdoctoral students PCM 577/17.08.2018 and by the Bulgarian National Science Fund under Project H 22/2 from 2018.

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Adjoin Operators for Three-Dimensional Continuity and Advection Equations

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We consider an explicit semi-Lagrangian algorithm for solving the three-dimensional continuity problem. To compute a numerical solution, we use a uniform space and time grids. The algorithm is based on splitting of three-dimensional problem into three one-dimensional problems. We determine a numerical solution of each one-dimensional problem as a function that is piecewise constant in sells at each time level. We use a local integral balance equation between two neighboring time levels, which follows from the Green's theorem. To find a numerical solution at next time level at the grid node, we consider integral over a space neighborhood of it. It produces the left-hand side of the algebraic equation. To get the right-hand side, we consider the integral of the numerical solution at the previous time level, which has already been determined. Thus, we get numerical scheme for three-dimensional continuity equation. We count all nodes of three-dimensional space grids by one enumerator and consider a matrix of numerical scheme. Each row in matrix corresponds to coincident time level. Adjoin matrix allows to find numerical solution of advection equation. It theoretically justified this help of adjoin operators. The proposed difference schemes are of first-order convergence that is confirmed by computational experiments.

Acknowledgements. The reported study was funded by Russian Foundation for Basic Research, Government of Krasnoyarsk Territory, Krasnoyarsk Regional Fund of Science to the research project "Numerical modeling of quasi-stable structures composed of multicomponent gas near a factory smokestack," project 18-41-243006.

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Convective Flow of Jeffrey Fluid past a Vertical Permeable Moving Plate

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In this article, we investigated the oscillatory MHD Free Convective Flow of Jeffrey Fluid past a vertical permeable moving plate. The formulated partial differential equations were solved using perturbation technique and velocity and temperature profiles are obtained. Numerical simulation was carried out using MATHEMATICA 10.3 to study the flow with some pertinent parameters such as and other parameters influence on the velocity and temperature profiles. It was observed that the variation of the pertinent parameters influences the flow profiles, as it leads to increasing velocity profile with temperature.

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Mathematical Model and Method for Calculating the Growth of Non-Metallic Inclusions in Multicomponent Melt

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In this contribution, a problem of the growing non-metallic inclusions in multicomponent melt is considered. The mathematical model of inclusion growth is represented as inverse problem for parabolic PDEs with the equations of boundary mass transfer and ODE describing the inclusion growth. In this problem, it is required to calculate the radius of growing spherical inclusion. To solve this problem, a numerical method based on the using of finite-difference equations and regularization technique is proposed. The application of regularization approach ensures the stability of computational scheme with respect to computational errors and errors of the initial data. The advantage of method is follows. Using the common approaches, we can calculate the radius at the early stage of growth only. This stage lasts for several seconds. The proposed method allows us to increase the observation time of the growing inclusion up to several minutes. The reliability of the method was verified by comparing the numerical results with the test functions.

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Vehicle Routing Problem with Time Windows

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The vehicle routing problem with time windows (VRPTW) is a famous problem in logistics that has many applications in real life. In this Problem, the objective is to define the minimized distance traveled of the several vehicles that start to move simultaneously from the depot and visit some customers within certain time frames. In this paper, an efficient optimization algorithm called football game algorithm (FGA) is proposed to solve VRPTW as an important NP-hard problem. FGA imitates the behavior of football players during a game to find the best positions to score a goal under the supervision of a team coach.The performance of the proposed FGA is validated against Solomon's VRPTW benchmarks. Experiment results confirm that FGA produces competitive results compared to several state-of-the-art algorithms in terms of the quality of the solutions. More precisely, the proposed algorithm obtains 17 and improves 12 best known solutions (BKSs) in the literature.

Keywords: Football Game Algorithm, Vehicle Routing Problem with Time Windows, Meta-heuristics, Solomon Benchmarks, Np-hard Problems

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On the Approximation of Estimation Problems for Controlled Systems

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In this talk, we consider problem of guaranteed estimation for controlled systems with discrete observations. For linear systems with continuous measurements, we reduce theirs to discrete ones and approximate the estimation procedure. Error bounds of the approximation are also found. Given a grid $t = t_0 \dots t_N$ on the time interval $[0, T]$, unknown controls $v(t)$ are changed for a sequence $v(t) = v_i$ on $(t_{i-1}, t_i]$. After that we come to the multistage system $x_i = f_i(x_{i-1}, v_i), \quad x_i \in R^n, \ i \in 1 : N$, with measurements $y_i = g_i(x_{i-1}, v_i) + w_i, \ y_i \in R^m$, and constraints $F_0(x_0) + \sum_{i=1}^{N} F_i(v_i, w_i)$, where F_i are non-negative functions. The problem is to give an analytical description of the set $\mathcal{W}_t$ compatible with measurements $y^t = \{y_1, \dots, y_t\}$ and consisting of all possible pairs (x_{t-1}, v_t). In addition to that one has to find the information set $\mathcal{V}_t = f_t(\mathcal{W}_t)$ which is the image of $\mathcal{W}_t$. These sets can be found by the recursive procedure. An application of this approach to linear differential systems of the form $\dot{x} = A(t)x + B(t)v$ with continuous observations $y(t) = G(t)x + w$ and quadratic integral constraints on unknown functions gives us the opportunity to obtain the approximation scheme for observation process. In this case the information sets are ellipsoids both for continuous and for discrete-time systems. The error bounds for ellipsoid parameters are obtained and the theorem is proved which asserts the convergence of the approximation. Various linear and nonlinear examples are considered including a model of two-element handlers, a motion in viscous medium, and the Lotka-Volterra model with incomplete information.

Acknowledgements. The work was partly supported by the Russian Foundation for Basic Research, Project No. 18-01-00544-a.

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Flexible Graphite as a Second-Grade Material

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Flexible graphite (hereinafter FG) is a unique nanostructured composite material with a temperature-independent high thermo-chemical durability, low coefficient of friction and high elastic properties. FG seals easily break-in without erosive affection on the contact metal surfaces, fit for a multifunctional usage in high-corrosive and high-reactive gas and fluid media. The FG o-ring seals and their packs have high reliability; they do not require additional pressurization during long-term usage and work at temperatures up to 560°C with pressures up to 40.0MPa. The influence of the FG microstructure can be taken into account by describing it as a linear isotropic second-gradient medium introduced in the works of R.A. Toupin and R.D. Mindlin. The scale parameter can be determined as a ratio of the distance between the graphite layers and the radius of the o-ring. The loading problem for the FG o-ring is solved in the gradient formulation. Friction conditions are established on the inner and outer radial surfaces of the ring. The friction coefficient is assumed to be small enough. On the bottom of the ring there are no axial displacements, on the top surface of the ring pressure is applied. The analytical solution of the loading problem under consideration will help to estimate the contribution of the gradient elastic moduli and the scale parameter into stress distribution and determine conditions of the gradient statement applicability. We suppose that the use of the gradient formulation is able to describe some phenomena that occur during the exploitation of FG o-rings. The work is dedicated to applying the gradient material models to solve actual engineering problems. Also, the results may be useful as an example of a problem solved in the second-gradient formulation.

Acknowledgements. The work is supported by the Russian Foundation for Basic Research (Grant No.17-41-590377_r_a).

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Noise-Induced Phantom Attractor in the Enzyme Kinetics

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We study the influence of random noise on the two dimensional model of enzymatic reaction. The model demonstrates high nonuniformity of deterministic phase portrait and sensitivity to parameters variations. A new dynamical phenomenon called a generation of phantom attractor is discovered when the model is forced by high intensity noise. It is an effect of shifting and localizing random states far from the deterministic attractor. The phenomenon is investigated using direct numerical simulations, probability density distribution and frequency analysis of stochastic oscillations.

Acknowledgements. The work was supported by Russian Science Foundation (No 16-11-10098).

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Order and Chaotic Regimes in 3D Goldbeter Model

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Three-dimensional nonlinear biochemical model proposed by Goldbeter is considered under the influence of random noise. In the deterministic case the model possesses a wide variety of dynamic regimes among which the periodic behavior, corresponding to limit cycle oscillations, combination of either two periodic oscillations or a limit cycle with an equilibrium and chaos. A transition of the system from order to chaos occurs through a cascade of period doubling bifurcations. It is studied how random noise can change the qualitative behavior of the model and form new dynamic regimes. An interaction of stochasticity and nonlinearity is discussed.

Acknowledgements. The work was supported by Russian Science Foundation (No 16-11-10098).

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Strong Topology on the Set of Persistence Diagrams

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We endow the set of persistence diagrams with the strong topology (the topology of countable direct limit of increasing sequence of bounded subsets considered in the bottleneck distance). The topology of the obtained space is described. Also, we prove that the space of persistence diagrams with the bottleneck metric has infinite asymptotic dimension in the sense of Gromov.

Statistical Study of the Influence of the Atmospheric Characteristics upon the Particulate Matter (PM10) Air Pollution in the City of Silistra, Bulgaria

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Air pollution by particulate matter with a particle diameter between 2.5 and 10μm (PM10) is going up recently in the entire Danube region, Bulgaria [1,2,5,6]. The sources of dust on the territory of Danube region, Bulgaria are industry, transport and domestic heating by solid fuel. PM10 levels for the entire Danube region in Bulgaria mark a significant increase during the autumn-winter period compared to the levels during the spring-summer period. For example in Ruse - a city belonging to this region the levels of PM10 are going up recently [5]. The biggest peak of PM10 levels for the autumn-winter period is usually observed in January months. It is in January that the number of days in which there is exceedance of the limit values of the PM10 levels is maximum observed.

Also recently it becomes clear that the mean value of the temperature in this region is slightly goes up for the last 40 years and it is bigger than the mean temperature for Bulgaria [5]. This could be interpreted as a proof for climate change and warming in Danube region. The presence of PM10 obviously affects and changes somehow the main atmospheric characteristics – temperature, atmospheric pressure and humidity and maybe there is a relationship between PM10 contamination levels and ambient air characteristics.

In our works [3,4,6] we examined in more details the influence of the atmospheric characteristics on the PM10 contamination during January months for one of the biggest cities in Bulgarian Danube region – Ruse. To understand better this relationship we provide a statistical analysis of ambient air PM10 contamination during winter periods [3,4]. Correlations between the measured PM10 values and the respective temperatures, atmospheric pressure and relative humidity measured for January months for different years were presented and commented there.

This paper is one continuation of our investigations of PM10 pollution for Danube region especially for another city of the region – Silistra. It is devoted to examine the PM10 pollution during winter and its relationship with atmospheric characteristics (temperature, atmospheric pressure and humidity). It presents a statistical analysis of the level of PM10 air pollution in Silistra using data from the official monitoring stations in the city. The measurements cover the period since 2011 till 2018. Descriptive statistics of PM10 and atmospheric characteristics-temperature, atmospheric pressure and relative humidity as well as linear regression modeling are presented and commented in the paper.

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Material Symmetry Conjugated Spin and Deformation Tensors for Orthotropic Media

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A constitutive material spin tensor in the case of purely elastic finite-strain deformation is introduced for a 3D orthotropic media using the minimizing principle applied to obtain the reloaded configuration of the material volume. This material spin explains the rotation of the orthonormal vector frame which coincides with the material symmetry axes in the initial configuration of the material volume and uniquely corresponds to a set of these axes in the current configuration although it does not coincide with the latter. The given definition is followed by the exact expression. This definition allows obtaining a new variant of decomposing any elastic finite-strain motion onto rigid and deformational parts and introducing the corotational rate associated with the material anisotropy. The latter is used for the formulation of the anisotropic rate-type elastic law in the current configuration based on the strain measure which does not belong to the Seth-Hill family. The introduced material symmetry rotation naturally accounts for the material symmetry and demonstrates the dependence of its rotation angle on the initial orientation of the anisotropy axes under finite-strain deformation. For the particular cases such as infinitesimal deformations or a higher material symmetry (more close to the isotropic one) the introduced material rotation tensor coincides with the conventional measures of rotation introduced in solid mechanics. By using the introduced material spin tensor for the rate-type elastic law, it becomes possible to take into account the material anisotropy in the finite-strain "hypo-elasticity" in contrast with all the known variants of the corotational rates. The introduced rate-type elastic law is appropriate for studying the finite-strain deformation of anisotropic materials as well as for modeling the process of crystal lattice rotation of subgrains in the course of intense plastic deformation.

Acknowledgements. The work was supported by the Russian Foundation for Basic Research (grant No. 17-41-590377).

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